Chapter 6

(2) $b \times 3b$

Find ratio of 1st 3 overtones to the fundamental

$$k = \sqrt{k_x^2 + k_y^2} \qquad f = \frac{c}{2}\sqrt{\left(\frac{n}{L_x}\right)^2 + \left(\frac{m}{L_y}\right)^2} \qquad L_x = b, \quad L_y = 3b$$

(m, n)

$$f = \frac{c}{2}\sqrt{\left(\frac{n}{b}\right)^2 + \left(\frac{m}{3b}\right)^2} = \frac{c}{2b}\sqrt{\left(\frac{n}{1}\right)^2 + \left(\frac{m}{3}\right)^2}$$

$$f_{11} = \frac{c}{2b}\sqrt{1^2 + 1/9} = \frac{c}{2b}\sqrt{\frac{10}{9}}$$

$$f_{21} = \frac{c}{2b}\sqrt{2^2 + (1/3)^2} = \frac{c}{2b}\sqrt{\frac{37}{9}}$$

$$f_{12} = \frac{c}{2b}\sqrt{1 + (2/3)^2} = \frac{c}{2b}\sqrt{13/9}$$

$$f_{13} = \frac{c}{2b}\sqrt{1 + (3/3)^2} = \frac{c}{2b}\sqrt{18/9}$$

$$f_{22} = \frac{c}{2b}\sqrt{2^2 + (2/3)^2} = \frac{c}{2b}\sqrt{\frac{40}{9}}$$

$$f_{14} = \frac{c}{2b}\sqrt{1^2 + (4/3)^2} = \frac{c}{2b}\sqrt{\frac{25}{9}}$$

$$f_{21} = \frac{c}{2b}\sqrt{\frac{37}{9}}\ \frac{2b}{c}\sqrt{9/10}\, f_{11} = \sqrt{37/10}\, f_{11} = 1.92 f_{11}$$

$$f_{13} = \frac{c}{2b}\sqrt{18/9}\ \frac{2b}{c}\sqrt{9/10}\, f_{11} = \sqrt{18/10}\, f_{11} = 1.34 f_{11} = f_{13}$$

$$f_{12} = \frac{c}{2b}\sqrt{\frac{13}{9}}\ \frac{2b}{c}\sqrt{9/10}\, f_{11} = \sqrt{13/10}\, f_{11} = 1.14 f_{11} = f_{12}$$

$$f_{14} = \frac{c}{2b}\sqrt{\frac{25}{9}}\ \frac{2b}{c}\sqrt{9/10}\, f_{11} = \sqrt{25/10}\, f_{11} = 1.58 f_{11} = f_{14}$$

#2

(4)

Aluminum $\rho = 2700\ kg/m^3$

$a = 2.5\ cm$ $\quad \rho_s = 2700\ kg/m^3 \left(1.2\times10^{-4}\ m\right) = 0.324\ kg/m^2$

$t = 0.012\ cm$

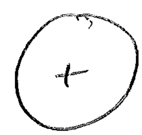
$T = 15{,}000\ N/m$ $\quad Z_{mn}(r,\theta,t) = A_{mn} J_m(k_{mn} r)\cos(m\theta + \epsilon_{mn})\cos(\omega_{mn} t + \phi_{mn})$

$c = \sqrt{\frac{T}{\rho_s}} = \sqrt{\frac{15{,}000\ N/m}{0.324\ kg/m^2}}$ $\quad k_{mn} = \frac{q_{mn}}{a}$

$c = 215\ m/s$

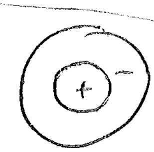
$m = 0 \quad n = 1$ $\quad Z_{01} = A_{01} J_0(k_{01} r)\cos(\epsilon_{01})\cos(\omega_{01} t + \phi_{01})$

$k_{01} = \frac{q_{01}}{a} = \frac{2.40}{2.5\times10^{-2}} = 96\ rad/m$



no nodal line $\quad f_{01} = \frac{\omega}{2\pi} = \frac{kc}{2\pi} = \frac{(96\ rad/m)(215\ m/s)}{2\pi} = \boxed{3285\ Hz} = f_{01}$

$k_{02} = \frac{q_{02}}{a} = \frac{5.52}{2.5\times10^{-2}} = 221\ rad/m$

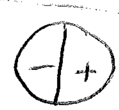


$f_{02} = \frac{(221\ rad/m)(215\ m/s)}{2\pi} = \boxed{7562\ Hz = f_{02}}$

nodal line circle at $k_{02}\cdot r = 2.40$

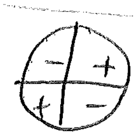
$r = \frac{2.40}{221} = \boxed{1.09\ cm}$ $\quad \cancel{r = \frac{221\ rad/m}{2.40}} =$

$k_{11} = \frac{q_{11}}{a} = \frac{3.83}{2.5\times10^{-2}} = 153\ rad/m$



$f_{11} = \frac{(153\ rad/m)(215\ m/s)}{2\pi} = \boxed{5235\ Hz = f_{11}}$

$k_{21} = \frac{q_{21}}{a} = \frac{5.14}{2.5\times10^{-2}} = 206\ rad/m$



$f_{21} = \frac{(206\ rad/m)(215\ m/s)}{2\pi} = \boxed{7049\ Hz = f_{21}}$

#3

(6) $S_{ST} = 1\times10^9\ Pa$

$S_{Al} = 2\times10^8\ Pa$

$D = 2\ cm$

$a = 1\ cm$

$\rho_{ST} = 7{,}700\ kg/m^3$

$\rho_{Al} = 2{,}700\ kg/m^3$

$$k_{01} = \frac{q_{01}}{a} = \frac{2.40}{1\times10^{-2}} = 240\ rad/m$$

$$C_{ST} = \sqrt{\frac{S_{ST}\cdot t}{\rho_{ST}\cdot t}} = \sqrt{\frac{1\times10^9\ Pa}{7700\ kg/m^3}} = 360\ m/s$$

$$C_{Al} = \sqrt{\frac{2\times10^8\ Pa}{2700\ kg/m^3}} = 272\ m/s$$

$$f_{ST} = \frac{\omega}{2\pi} = \frac{ck}{2\pi} = \frac{(360\ m/s)(240\ rad/m)}{2\pi} = \boxed{13.8\ kHz}$$

$$f_{Al} = \frac{\omega}{2\pi} = \frac{ck}{2\pi} = \frac{(272\ m/s)(240\ rad/m)}{2\pi} = \boxed{10.4\ kHz}$$

---

(8) $D = 50\ cm$

$a = 0.25\ m$

$\rho_s = 1.0\ kg/m^2$

$T = 10{,}000\ N/m$

(a)
$$k_{01} = \frac{q_{01}}{a} = \frac{2.40}{0.25\ m} = 9.6\ rad/m$$

$$c = \sqrt{\frac{T}{\rho_s}} = \sqrt{\frac{10{,}000\ N/m}{1\ kg/m^2}} = 100\ m/s$$

$$f = \frac{\omega}{2\pi} = \frac{ck}{2\pi} = \frac{(100\ m/s)(9.6\ rad/m)}{2\pi} = \boxed{153\ Hz}$$

(b) Backing Vessel

$p = 1\times10^5\ Pa$

$\gamma = 1.4$

$$B = \frac{\gamma P_0}{T V_0}\pi a^4$$

$$V_0 = \frac{1}{2}\left(\frac{4}{3}\pi a^3\right) = \frac{1}{2}\left(\frac{4}{3}\pi (.25)^3\right) = 0.03272\ m^3$$

$$B = \frac{(1.4)(1\times10^5\ Pa)\ \pi\ (0.25)^4}{(10{,}000\ N/m)(0.03272\ m^3)} = 5.25$$

$$k_{mod} = \frac{3.02}{0.25\ m} = 12.08\ rad/m$$

$$f = \frac{(100\ m/s)(12.08\ rad/m)}{2\pi} = \boxed{192\ Hz}$$

#4

(ii) $D = 30\,mm$ $\quad a = 1.5 \times 10^{-2}\,m$ $\quad k_{01} = \frac{2.40}{1.5\times10^{-2}\,m} = 160\ rad/m$

$t = 0.02\,mm$

$S_{Al} = 2\times10^{8}\,Pa$

$\rho_{Al} = 2700\ kg/m^3$

$$c = \sqrt{\frac{T}{\rho_s}} = \sqrt{\frac{S}{\rho}} = \sqrt{\frac{2\times10^{8}\,Pa}{2700\,kg/m^3}} = 272\ m/s$$

$$f = \frac{\omega}{2\pi} = \frac{ck}{2\pi} = \frac{(272\,m/s)(160\,rad/m)}{2\pi} = \boxed{6926\ Hz}$$

$$T = S_{Al}\cdot t = (2\times10^{8}\,Pa)(2\times10^{-5}\,m) = 4\times10^{3}\ N/m$$

$V = 2.5\times10^{-7}\,m^3$

$P = 1\times10^{5}\,Pa$

$\gamma = 1.4$

$$B = \frac{\gamma P_0 \pi a^4}{T V_0}$$

$$B = \frac{(1.4)(1\times10^{5}\,Pa)\pi(1.5\times10^{-2}\,m)^4}{(4\times10^{3}\,N/m)(2.5\times10^{-7}\,m^3)} = 22.3$$

Estimate new q value

$q_{B=5} = 3.02$ $\quad q_{B=10} = 3.485 \xrightarrow[+0.465]{} q_{B=20} = 3.95$

Then $k = \frac{3.95}{1.5\times10^{-2}} = \cancel{2.6}\,263\ rad/m$

$$f = \frac{ck}{2\pi} = \frac{(272\,m/s)(263\,rad/m)}{2\pi} = \boxed{11.4\ kHz}$$

frequency increase $\boxed{1.6}$ of un backed frequency

#5

(13) $D = 4cm$

$b = t = 0.18 mm$

$\rho_{ST} = 7700 \ kg/m^3$

$E_{ST} = 195 \times 10^9 \ Pa$

$\mu_{ST} = 0.28$

$$f = \frac{\kappa K^2}{2\pi}\sqrt{\frac{E}{\rho(1-\mu^2)}}$$

(a)

$$\kappa = \frac{b}{\sqrt{12}} = \frac{1.8 \times 10^{-4}}{\sqrt{12}} = 5.196 \times 10^{-5} \ m$$

$$K = \frac{3.20}{a} = \frac{3.20}{2 \times 10^{-2} m} = 160 \ m^{-1}$$

$$f = \frac{(5.196 \times 10^{-5} m)(160 m^{-1})^2}{2\pi}\sqrt{\frac{195 \times 10^9 Pa}{7700 \frac{kg}{m^3}(1-0.28^2)}}$$

$$\boxed{f = 1.11 \ kHz}$$

(b) double Thickness

~~Freq~~ κ depends on thickness directly

Therefore $\boxed{\text{2x frequency when 2x thickness}}$

(c) $a \times 1.5$

K depends inversely with a

$$a = a \times 1.5 \rightarrow K = \frac{1}{1.5} K_0$$

$$f = \left(\frac{1}{1.5}\right)^2 f_0 = 0.44 f_0 = \frac{100}{225} f_0 = \boxed{\frac{4}{9} f_0}$$

---

(14) $D = 25cm$

$t = 0.55 mm$

$\rho_{ST} = 7700 \ kg/m^3$

$E_{ST} = 195 \times 10^9 \ Pa$

$\mu_{ST} = 0.28$

$$f = \frac{(1.59 \times 10^{-4} m)(25.6 m^{-1})^2}{2\pi}\sqrt{\frac{195 \times 10^9 Pa}{(7700 kg/m^3)(1-0.28^2)}}$$

$$\boxed{f = 86.9 \ Hz}$$

$$\kappa = \frac{b}{\sqrt{12}} = \frac{5.5 \times 10^{-4} m}{\sqrt{12}} = 1.59 \times 10^{-4} m$$

$$K = \frac{3.20}{a} = \frac{3.20}{0.125 m} = 25.6 \ m^{-1}$$