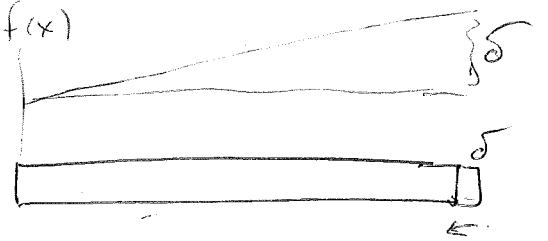
Chapter 5



(2) $L = 0.60\,m$

fixed at $x=0$

free at $x=L$

steel

$f_0 =$

$$c = \sqrt{\frac{E}{\rho}} = \sqrt{\frac{195\times10^9\,Pa}{7700\,kg/m^3}} = 5.03\times10^3\,m/s$$

from B.C.,

$$kL = \frac{\pi}{2}(2n-1) \qquad k_1 = \frac{\pi}{2L} = \frac{\pi}{2(0.6m)} = 2.62\ rad/m$$

$$\lambda = \frac{2\pi}{k} = \frac{2\pi}{2.62\ rad/m} = 2.4\,m$$

$$f = \frac{c}{\lambda} = \frac{\omega}{2\pi} = \frac{kc}{2\pi} = \frac{(2.62\ rad/m)(5.03\times10^3\,m/s)}{2\pi} = 2.10\times10^3\,Hz$$

2.10 kHz

Solution

Fixed-Free

~~$\xi = A_n \sin(k_n x) e^{i\omega t}$~~

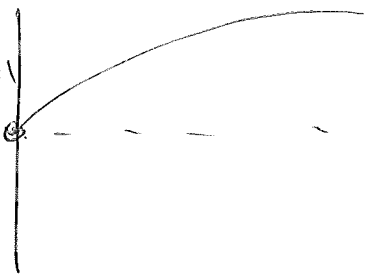
$$k_n = \frac{\pi}{2}\frac{(2n-1)}{L}$$

$$\xi_n = \sin(k_n x)(A_n \cos\omega_n t + B_n \sin\omega_n t)$$

Displaced only (no initial velocity)

$v_i = 0 \;\therefore\; B_n = 0$

Fixed-Free



At $t=0$

~~$\xi(0) = A_n \sin(k_n x)$~~

$t=0s \qquad \xi(0) = \sum A_n \sin(k_n x)\cos(0) = f(x)$

$f(x) = mx + b$

$$f(x) = \frac{\delta}{L}\cdot x$$

$$A_n = \frac{2}{L}\int_0^L \left(\frac{\delta}{L}x\right)\sin k_n x\,dx$$

Fixed-Fixed

#2

③ steel

x=0 - free

$A = 2.0\times10^{-4}\ m^2$

$L = 0.35\ m$

$m_L = 0.20\ kg$

$\rho_{ST} = 7700\ kg/m^3$

④ $\frac{\tan kL}{kL} = \frac{m}{m_b} = \frac{-0.20\ kg}{0.539\ kg} = -0.3710$

kL	tan kL	-0.371 kL
2.4 ~~2.6~~	-0.9160	-0.8904
2.45	-0.8280	-0.9090
2.41	-0.8978	-0.8941
2.42	-0.8799	-0.8978

$kL = 2.41$

$k = \frac{2.41}{0.35} = 6.89$

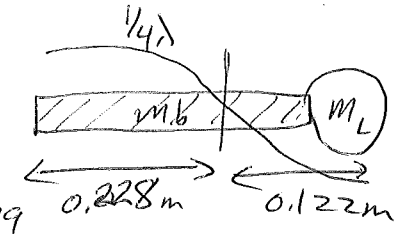
$m_b = \rho_{ST} \cdot A \cdot L = (7700 kg/m^3)(2.0\times10^{-4} m^2)(0.35 m) = 0.539\ kg$

$k = 6.89$

$f = \frac{\omega}{2\pi} = \frac{ck}{2\pi} = \frac{(5032\ m/s)(6.89\ rad/m)}{2\pi} = \boxed{5.5\ kHz}$

$c = \sqrt{\frac{E}{\rho}} = \sqrt{\frac{195\times10^9 Pa}{7700 kg/m^3}}$

$= 5,032\ m/s$



⑤ clamp point

$k = 6.89$

$\lambda = \frac{2\pi}{k} = \frac{2\pi}{6.89} = 0.912\ m$

first node at $\frac{\lambda}{4} = \frac{0.912\ m}{4} = \boxed{0.228\ m}$

Remaining bar is 0.122 m

⑧

⑥ 1st overtone

let x=0 A=1

$\psi = A\cos(kx)$

$\psi = \cos(kx)$

$\psi(0) = 1$

$\psi(0.106) = 0$

$\psi(0.318) = 0$

$\psi(0.35) = 0.460$

kL	tan kL	-0.371 kL
5.5	-0.9956	-2.0405
5.4	-1.218	-2.003
5.3	-1.501	-1.9663
5.2	-1.886	-1.929
5.19	-1.932	-1.925

$kL = 5.19$

$k = \frac{5.19}{0.35} = \boxed{14.83}$

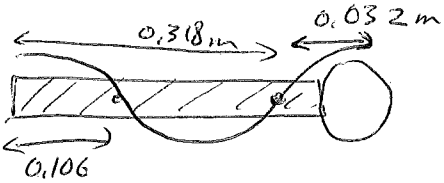
$\lambda = \frac{2\pi}{k} = \frac{2\pi}{14.83} = 0.424\ m$

first node at $\frac{\lambda}{4} = \frac{0.424\ m}{4} = 0.106\ m$

2nd node at $\frac{3\lambda}{4} = 0.318\ m$

$\frac{free}{tool} = \frac{1}{0.460}$

$\boxed{\frac{free}{tool} = 2.17}$



#3

(9) $v = \sqrt{K\omega c}$

~~when~~ will set what freq. $c = c_p = v$

$D = 0.005\,m$

steel rod

$$K = \frac{a}{2} = \frac{0.0025\,m}{2} = 1.25 \times 10^{-3}\,m$$

$[\omega] = rad/s$

$[c] = m/s$

$[K] = m$

$$[v] = \sqrt{m \cdot \frac{rad}{s} \cdot \frac{m}{s}} = \sqrt{\frac{m^2}{s^2}} = m/s$$

$$c = \sqrt{K\omega c} \Rightarrow c^2 = K\omega c$$

$$c = K\omega$$

$$c = \sqrt{\frac{E}{\rho_s}} = \sqrt{\frac{195 \times 10^9\,Pa}{7700\,kg/m^3}}$$

$$c = 5032\,m/s$$

$$\omega = \frac{c}{K} = \frac{5032\,m/s}{1.25 \times 10^{-3}\,m}$$

$$= 4.03 \times 10^6\,rad/s$$

$$\boxed{f = 641\,kHz}$$

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(10)

$a = 0.010\,m$

$L = 0.4\,m$

$\rho_{AL} = 2700\,kg/m^3$

$E_{AL} = 71 \times 10^9\,Pa$

$c_{AL} = 5128\,m/s$

$$K = \frac{a}{2} = \frac{0.010\,m}{2} = 0.005\,m$$

$$f = \zeta^2 \frac{\pi K c}{8L^2} = \frac{(3.0112)^2 \pi (0.005\,m)(5128\,m/s)}{8(0.4\,m)^2}$$

(b) $\boxed{f = 571\,Hz}$

$$\omega = 2\pi f = 2\pi(571\,Hz) = 3588\,rad/s$$

$$v = \sqrt{K\omega c} = \sqrt{(0.005\,m)(3588\,rad/s)(5128\,m/s)} = \boxed{303\,m/s}$$

$\cos x = \frac{e^{ix} + e^{-ix}}{2}$

$\sin x = \frac{e^{ix} - e^{-ix}}{2}$

#4 (8) continued

$$\psi_{(x)} = A\cosh\frac{\omega x}{v} + B\sinh\frac{\omega x}{v} + C\cos\frac{\omega x}{v} + D\sin\frac{\omega x}{v}$$

free-free $\therefore$ $A=C$ $B=D$

$$\psi = A\cosh\frac{\omega x}{v} + B\sinh\frac{\omega x}{v} + A\cos\frac{\omega x}{v} + B\sin\frac{\omega x}{v}$$

$$\psi(0) = \psi(L)$$

$$A\cosh(0) + B\sinh(0) + A\cos(0) + B\sin(0) = A\cosh\left(\frac{\omega L}{v}\right) + B\sinh\left(\frac{\omega L}{v}\right) + A\cos\left(\frac{\omega L}{v}\right) + B\sin\left(\frac{\omega L}{v}\right)$$

$$A + A = A\cosh\left(\frac{\omega L}{v}\right) + B\sinh\left(\frac{\omega L}{v}\right) + A\cos\left(\frac{\omega L}{v}\right) + B\sin\left(\frac{\omega L}{v}\right)$$

$$2A - A\cosh\left(\frac{\omega L}{v}\right) - A\cos\left(\frac{\omega L}{v}\right) = B\sinh\left(\frac{\omega L}{v}\right) + B\sin\left(\frac{\omega L}{v}\right)$$

$$\frac{A}{B} = \frac{\sinh\left(\frac{\omega L}{v}\right) + \sin\left(\frac{\omega L}{v}\right)}{2 - \cosh\left(\frac{\omega L}{v}\right) - \cos\left(\frac{\omega L}{v}\right)}$$

$\frac{\omega L}{v} = 4.737$

$\frac{\omega L}{2v} = 2.368$

$$\psi(L/2) = 2.5\text{ cm} = A\cosh\left(\frac{\omega L}{2v}\right) + B\sinh\left(\frac{\omega L}{2v}\right) + A\cos\left(\frac{\omega L}{2v}\right) + B\sin\left(\frac{\omega L}{2v}\right)$$

$$\frac{A}{B} = \frac{57.04 + -0.9997}{2 - 57.05 - 0.0246} = -1.018$$

$\sinh\left(\frac{\omega L}{v}\right) = 57.04$

$\sinh\left(\frac{\omega L}{2v}\right) = 5.294$

$\cosh\left(\frac{\omega L}{v}\right) = 57.05$

$\cosh\left(\frac{\omega L}{2v}\right) = 5.387$

$\sin\left(\frac{\omega L}{2}\right) = -0.9997$

$\sin\left(\frac{\omega L}{2v}\right) = 0.6984$

$\cos\left(\frac{\omega L}{2}\right) = 0.0246$

$\cos\left(\frac{\omega L}{2v}\right) = -0.7158$

$2.5\text{ cm} = -1.018\,B(5.387) + B(5.294) + (-1.018)B(-0.7158) + B(0.6984)$

$2.5\text{ cm} = 1.237\,B$

$B = 2.02\text{ cm}$

$A = -2.06\text{ cm}$

$\psi(0) = 2A = -4.12\text{ cm} = \psi(0)$

$\psi(L) = (-2.06)(57.05) + (2.02)(57.04) + (-2.06)(0.0246) + (2.02)(-.9997)$

$\psi(L) = -4.37\text{ cm} = \psi(L)$