Homework Ch 4

Euler's Relationship
$e^{ix} = \cos x + i \sin x$

#1

a) $f(x-ct)$
b) $\ln[f(x-ct)]$
c) $A(ct-x)^3$
d) $\sin[A(ct-x)]$

sub into $\Longrightarrow$ $\frac{\partial^2 p}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2}$

should be consistent

#4

$\delta = 0.05\ g/cm$
$y = 4\cos(5t-3x)$

$y = A\cos(kx - \omega t + \phi)$

$c^2 = \frac{T}{\delta}$

a) $\boxed{A = 4}$

$c = \omega/k = 5/3 = \boxed{1.67\ cm/s = c}$

$\omega = 5$ $\quad f = 5/2\pi = \boxed{0.8\ Hz = f}$

$\lambda = 2\pi/3 = \boxed{2.1\ cm = \lambda}$

$\boxed{k = 3}$

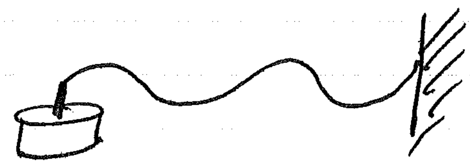
b) $\dot{y} = -5 \cdot 4 \sin(5t-3x)$
$= -20\sin(5t-3x)$

$\boxed{\dot{y}(0,0) = 0\ cm/s}$

Chapter 4

(2)
(a) $B(ct - x^2)$ No
(b) $C(ct - c)t$ No
(c) $A + B\sin(ct + x)$ yes
(d) $A\cos^2(ct - x) + B\sin(ct + x)$ yes

(7)
$\delta = 0.02\ kg/m$
$T = 8N$
$L = 0.52\ m$
Node spacing $0.1\ m$
$A = 0.022\ m$



~~What is f & f~~
Find amplitude & freq of driving force

$\lambda = 2(0.1m) = 0.2m$
$k = 2\pi/\lambda = 2\pi/0.2 = 10\pi$
$c = \sqrt{T/\delta} = \sqrt{\dfrac{8N}{0.02kg/m}} = 20\ m/s$
$f \cdot \lambda = c \quad \therefore f = c/\lambda = \dfrac{20m/s}{0.2m} = \boxed{100\ Hz}$

$Z_s = -i\delta c \cot kL$ ~~$= -i(0.02kg/m)(20m/s)\cot(10\pi$~~

$= -i(0.02kg/m)(20m/s)\cot(10\pi\ 0.52m)$
$\boxed{Z_s = -i\ 0.551}$

$v = \dfrac{dy}{dt} = \dfrac{d}{dt}(A\cos(\omega t - kx)) = -\omega A\sin(\omega t - kx)$

Since
$Z_s = \dfrac{Fe^{i\omega t}}{v}$ $\quad v = 2\pi f A = 13.8$

Ignoring phase $\quad Z_s = \dfrac{F}{v}$

$F = Z_s \cdot v = Z_s \cdot \omega A$
$= (0.551)2\pi(100Hz)(0.022m) = \boxed{7.62N}$

(9) Consider ρ_L, c, L, f, + T

(a) double length

$$c' = \sqrt{\frac{T'}{\rho_L'}} = \sqrt{\frac{T}{\rho_L}} = c \qquad \therefore c' = c$$

$$f' = \frac{c'}{\lambda'} = \frac{c}{2\lambda} = \frac{1}{2}\frac{c}{\lambda} = \frac{1}{2}f \qquad \therefore f' = \frac{1}{2}f$$

(b) double ρ_L

$$c' = \sqrt{\frac{T'}{\rho_L'}} = \sqrt{\frac{T}{2\rho_L}} = \frac{1}{\sqrt{2}}\sqrt{\frac{T}{\rho_L}} = \frac{1}{\sqrt{2}}c \qquad \therefore c' = \frac{c}{\sqrt{2}}$$

$$f' = \frac{c'}{\lambda'} = \frac{c}{\sqrt{2}\lambda} = \frac{1}{\sqrt{2}}\frac{c}{\lambda} = \frac{1}{\sqrt{2}}f \qquad \therefore f' = f/\sqrt{2}$$

(c) double cross-sectional area

(assume $\rho_L \propto A$ Then $\rho_L' = 2\rho_L$

→ will be same as (b)

otherwise $c' = c$

$f' = c$

(d) Tension is $1/2$

$$c' = \sqrt{\frac{T'}{\rho_L'}} = \sqrt{\frac{T}{2\rho_L}} = \frac{1}{\sqrt{2}}\sqrt{\frac{T}{\rho_L}} = c/\sqrt{2} \qquad \therefore c' = \frac{c}{\sqrt{2}}$$

$$f' = \frac{c'}{\lambda'} = \frac{c}{\sqrt{2}\lambda} = \frac{1}{\sqrt{2}}f \qquad \therefore f' = f/\sqrt{2}$$

(e) diameter doubled

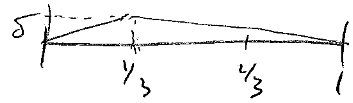
assume $\rho_L \propto D^2$ Then $\rho_L' = 4\rho_L$

$$c' = \sqrt{\frac{T'}{\rho_L'}} = \sqrt{\frac{T}{4\rho_L}} = \frac{1}{2}\sqrt{T/\rho_L} = \frac{1}{2}c \qquad \therefore c' = c/2$$

$$f' = \frac{c'}{\lambda'} = \frac{c}{2\lambda} = \frac{1}{2}f \qquad \therefore f' = f/2$$

#10 L plucked at $L/3$ displacement δ

Find fundamental & 1st 3- harmonic overtones.



$$A_n = \frac{2}{L}\int_0^L y_0(x)\sin k_n x\,dx \qquad k_n = \frac{n\pi}{L}$$

$$A_n = 2\int_0^{1/3} 3\delta x \sin(n\pi x)dx + 2\int_{1/3}^{1} \left(\frac{3\delta}{2} - \frac{3\delta}{2}x\right)\sin(n\pi x)\,dx$$

$$A_n = 6\delta\int_0^{1/3} x\sin(n\pi x)\,dx + \frac{6\delta}{2}\int_{1/3}^{1}(1-x)\sin(n\pi x)\,dx$$

$$A_n = 6\delta\left(\frac{\sin(n\pi x)}{n^2\pi^2} - \frac{x\cos(n\pi x)}{n\pi}\right)\Bigg|_0^{1/3} + \frac{6\delta}{2}\left(-\frac{\cos(n\pi x)}{n\pi} - \frac{\sin(n\pi x)}{n^2\pi^2} + \frac{x\cos(n\pi x)}{n\pi}\right.$$

$$A_1 = 6\delta\left(\frac{\sin\pi/3}{\pi^2} - \frac{\frac{1}{3}\cos\pi/3}{\pi} - 0 + 0\right) + \frac{6\delta}{2}\left(-\frac{\cos\pi}{\pi} - \frac{\sin\pi}{\pi^2} + \frac{1\cdot\cos\pi}{\pi} + \frac{\cos\pi/3}{\pi} + \frac{\sin\pi/3}{\pi^2} - \frac{\frac{1}{3}\cos\pi/3}{\pi}\right)$$

$$= 6\delta\left(\frac{\sqrt{3}}{2\pi^2} - \frac{1}{6\pi}\right) + \frac{6\delta}{2}\left(\frac{1}{\pi} - 0 - \frac{1}{\pi} + \frac{1}{2\pi} + \frac{\sqrt{3}}{2\pi^2} - \frac{1}{6\pi}\right)$$

$$= \frac{6\delta}{2\pi}\left[\frac{\sqrt{3}}{\pi} - \frac{1}{3} + 1 - 1 + \frac{1}{2} + \frac{\sqrt{3}}{2\pi} - \frac{1}{6}\right] = \boxed{0.7897} = A_1$$

$$\boxed{0.1974} = A_2$$

$$\boxed{0} = A_3$$

$$\boxed{-0.04935} = A_4$$

(13) $y = 3\sin(\pi x/4)\sin 2t$

(a) $L = 36\,cm$

$\delta = 0.1\,g/cm \left(\frac{1\,kg}{1000\,g}\right)\left(\frac{100\,cm}{1\,m}\right) = 0.01\,kg/m$

From the equation $\omega = 2\,rad/s$

$k = \pi/4\,rad/cm$

$\therefore f = \frac{\omega}{2\pi} = \frac{2}{2\pi} = \boxed{\frac{1}{\pi}\,Hz = f}$

$\boxed{k = \pi/4}\,rad/cm \rightarrow k = \frac{\pi}{4}\frac{rad}{cm}\,\frac{100\,cm}{1\,m} = \boxed{25\pi\,rad/m}$

$c = \omega/k = \frac{2}{25\pi} = \cancel{\frac{8}{\pi}}\ \boxed{\frac{2}{25\pi}\,m/s}$

(b) $x = 18\,cm \quad A = 3\sin(\pi 18/4) = 3\,cm$

$x = 36/4 = 9\,cm \quad A = 3\sin(\pi 9) = 0$

$x = 36/3 = 12\,cm \quad A = 3\sin(\pi 12) = 0$

(c) $\frac{dE}{dx} = \frac{\omega_n^2\delta}{2}(A_n^2 + B_n^2)\sin^2 k_n x$

at $x = 18\,cm$ $\quad \frac{dE}{dx} = \frac{2^2(0.01\,kg/m)}{2}((.03)^2) = \boxed{1.8\times10^{-5}}$

total Energy

$E_n = \frac{m}{4}\omega_n^2(A_n^2 + B_n^2)$

$= \frac{(0.36\,m)(0.01\,kg/m)}{4}(2\,rad/s)^2(.03\,m)^2$

$\boxed{E_n = 3.2\times10^{-6}\,J}$