Chapter 2

(2) $f = 100\,Hz$

$v = 330\,m/s$

$\omega = 2\pi f = 2\pi(100\,Hz) = 628\,rad/s$

$v = f \cdot \lambda \rightarrow \lambda = \frac{v}{f} = \frac{330\,m/s}{100\,Hz} = \boxed{3.3\,m}$

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(3) $\gamma = 1.402$

$\rho = 1.293\,\frac{kg}{m^3}$

$p = 101.2\,kPa = 1.012 \times 10^5\,Pa$

$T = 0^\circ C$

$c = \sqrt{\frac{\gamma p}{\rho}} = \sqrt{\frac{(1.402)(1.012 \times 10^5\,Pa)}{1.293\,kg/m^3}}$

$\boxed{c = 331.2\,m/s}$

$\cancel{c = 331\,m/s}$

or

$c = \sqrt{\gamma R T} = \sqrt{(1.402)\left(287\,\frac{Nm}{kg\,K}\right)(273.2\,K)}$

$\boxed{c = 331.6}$

Both give similar results.
Depending on which one + how you round, the answer is 331 or 332 m/s

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(6) $\rho = 8.5\,g/cm^3$

$c = 4000\,m/s$

$\rho = \left(\frac{8.5\,g}{cm^3}\right)\left(\frac{100\,cm}{1\,m}\right)^3\left(\frac{1\,kg}{1000\,g}\right)$

$\rho = 8500\,kg/m^3$

Find Young's Modulus

$c = \sqrt{\frac{E}{\rho}} \rightarrow c\rho^2 = E$

convert density to kg/m², then find E

$E = c\rho^2 = (4000\,m/s)(8500\,kg/m^3)^2$

$= 2.89 \times 10^{11}\,Pa$

$\boxed{E = 289\,GPa}$

(8)

$$c(P,t) = 1402.7 + 488t - 482t^2 + 135t^3 + (15.9 + 2.8t + 2.4t^2)(P_a/100)$$

$t = 0.01\,T$

P_a - in bars

Find c at $20°C$ + 1 bar $\therefore$ $\boxed{t = 0.01(20) = 0.20}$ $\boxed{P_a = 1}$

so $$c(1\text{ bar}, 0.20) = 1402.7 + 488(0.2) - 482(0.2)^2 + 135(0.2)^3 + (15.9 + 2.8(0.2) + 2.4(0.2)^2)(1/100)$$

$$c = 1482.1 + (16.56)\left(\tfrac{1}{100}\right) = \boxed{1482 \text{ m/s}}$$

$f = 200\text{ Hz}$

What is the wavelength?

$$\lambda \cdot f = c \longrightarrow \lambda = \frac{c}{f} = \frac{1482 \text{ m/s}}{200 \text{ Hz}}$$

$$\boxed{\lambda_w = 7.41 \text{ m}}$$

What is the wavelength through air?

$$\lambda \cdot f = c \rightarrow \lambda = \frac{c}{f} = \frac{343 \text{ m/s}}{200 \text{ Hz}}$$

$$\boxed{\lambda_A = 1.72 \text{ m}}$$

(9) why is $\underline{p}$ + $\underline{\rho}$ out of phase with $\underline{\underline{u}}$?

From the adiabatic process of an ideal gas, $p\rho^{-\gamma} = \text{const}$.
Another way of writing this is $\frac{p}{\rho^{\gamma}} = \text{const}$. From this form it is clear that if pressure increases, then density increases. They are directly proportional. Therefore, when you perturb pressure the density is perturbed "in phase" with it.

From the 1 dimensional form of the continuity equation we have
$$\frac{\partial(\rho \cdot u)}{\partial x} = \frac{\partial \rho}{\partial t}$$

~~If we look at the left hand side we see that the spatial variation applies to the product of density and velocity. If we focus on this relationship we see that for~~

For the sake of argument, let's say that the density does not have a time dependence. Then we would have
$$\frac{\partial(\rho \cdot u)}{\partial x} = 0$$
This would be true if $\rho \cdot u = \text{constant}$.
Now if density were to increase slightly, then in order for to product to be constant, the velocity would decrease. There is an inverse relationship between density + velocity. Therefore, they will be out of phase with each other. Likewise velocity will be out of phase with pressure.

(12) show $y(x,t)$ is a solution to the wave equation.

The ~~1D~~ ~~PDE~~ wave equation is $\dfrac{\partial^2 y}{\partial x^2} = \dfrac{1}{c^2}\dfrac{\partial^2 y}{\partial t^2}$

Preform the appropriate partial derivatives on each of the 4 terms of $y(x,t)$

$\underline{A_1 \cos(x-ct)}$:

$$\frac{\partial A_1 \cos(x-ct)}{\partial x} = -A_1 \sin(x-ct) \qquad \frac{\partial^2 A_1 \cos(x-ct)}{\partial x^2} = -A_1 \cos(x-ct)$$

$$\frac{\partial A_1 \cos(x-ct)}{\partial t} = -A_1 \sin(x-ct)(-c) \qquad \frac{\partial^2 A_1 \cos(x-ct)}{\partial t^2} = -A_1 \cos(x-ct)(-c)^2$$

sub into wave equation

$$-A_1 \cos(x-ct) = \frac{1}{c^2}\left(-A_1 \cos(x-ct)\right)(-c)^2$$

$$-A_1 \cos(x-ct) = -A_1 \frac{\cos(x-ct)}{c^2} c^2 = -A_1 \cos(x-ct)$$

Equation is true ∴ $\boxed{\text{It is a solution}}$

$\underline{B_1 \sin(x+ct)}$:

$$\frac{\partial B_1 \sin(x+ct)}{\partial x} = B_1 \cos(x+ct) \qquad \frac{\partial^2 B_1 \sin(x+ct)}{\partial x^2} = -B_1 \sin(x+ct)$$

$$\frac{\partial B_1 \sin(x+ct)}{\partial t} = B_1 \cos(x+ct)(c) \qquad \frac{\partial^2 B_1 \sin(x+ct)}{\partial t^2} = -B_1 \sin(x+ct)(c)^2$$

sub into wave equation

$$-B_1 \sin(x+ct) = \frac{1}{c^2}\left(-B_1 \sin(x+ct)(c^2)\right)$$

$$-B_1 \sin(x+ct) = -B_1 \sin(x+ct)$$

Equation is true ∴ $\boxed{\text{It is a solution}}$

$A_2 \cos^2 2(x-ct)$: ~~Do a trig substitute~~ Use a trig identity. We get $A_2\left(\frac{1}{2}+\frac{\cos(4(x-ct))}{2}\right)$

$$\frac{\partial A_2\cos^2 2(x-ct)}{\partial x} = \frac{-A_2}{2}\sin(4(x-ct))(4)$$

$$\frac{\partial^2 A_2\cos^2 2(x-ct)}{\partial x^2} = \frac{-A_2}{2}\cos(4(x-ct))(4)^2$$

$$\frac{\partial A_2\cos^2 2(x-ct)}{\partial t} = -\frac{A_2}{2}\sin(4(x-ct))(-4c)$$

$$\frac{\partial^2 A_2\cos^2 2(x-ct)}{\partial t^2} = -\frac{A_2}{2}\cos(4(x-ct))(-4c)^2$$

sub into wave equation

$$-\frac{A_2}{2}\cos(4(x-ct))(4)^2 = \frac{1}{c^2}\frac{-A_2}{2}\cos(4(x-ct))(-4c)^2$$

$$-\frac{A_2}{2}\cos(4(x-ct))(16) = \frac{-A_2}{2}\cos(4(x-ct))(16)$$

Equation is true $\therefore$ It is a solution

$B_2 \sin^2 3(x+ct)$: Use a trig identity. We get $B_2\left(\frac{1}{2}-\frac{\cos(6(x+ct))}{2}\right)$

$$\frac{\partial B_2\sin^2 3(x+ct)}{\partial x} = \frac{+B_2\sin(6(x+ct))(6)}{2}$$

$$\frac{\partial^2 B_2\sin^2 3(x+ct)}{\partial x^2} = \frac{B_2\cos(6(x+ct))(6)^2}{2}$$

$$\frac{\partial B_2\sin^2 3(x+ct)}{\partial t} = \frac{B_2\sin(6(x+ct))(6c)^2}{2}$$

$$\frac{\partial^2 B_2\sin^2 3(x+ct)}{\partial t^2} = \frac{B_2\cos(6(x+ct))(6c)^2}{2}$$

sub into wave equation

$$\frac{B_2\cos(6(x+ct))(6)^2}{2} = \frac{1}{c^2}\frac{B_2\cos(6(x+ct))(6c)^2}{2}$$

$$\frac{B_2\cos(6(x+ct)(36)}{2} = \frac{B_2\cos(6(x+ct))(36)}{2}$$

Equation is true $\therefore$ It is a solution

The left propagating wave is $B_1\sin(x+ct) + B_2\sin^2 3(x+ct)$

The right propagating wave is $A_1\cos(x-ct) + A_2\cos^2 2(x-ct)$

13 $\rho = \rho_o e^{i(x-ct)}$

For adiabatic process $p\rho^{-\gamma} = \text{const} = P_o \rho_o^{-\gamma}$

Following the development ~~for~~ of differentiating this equation we get equation 2.23 in the book

$$\frac{1}{P_o}\frac{\partial p}{\partial t} = \frac{\gamma}{\rho_o}\frac{\partial \rho}{\partial t}$$

Now $\frac{\partial \rho}{\partial t} = \frac{\partial}{\partial t}\left(\rho_o e^{i(x-ct)}\right) = -ic\rho_o e^{i(x-ct)}$ #1

taking equation 2.23 and isolate ∂p, we get

$$\frac{\partial p}{\partial t} = \frac{\gamma P_o}{\rho_o}\frac{\partial \rho}{\partial t}$$

$$\partial p = \frac{\gamma P_o}{\rho_o}\left(\frac{\partial \rho}{\partial t}\right)\partial t$$

substitute #1 in for $\frac{\partial \rho}{\partial t}$ and integrate both sides

$$\partial p = \frac{\gamma P_o}{\rho_o}\left(-ic\rho_o e^{i(x-ct)}\right)\partial t$$

$$\int \partial p = -ic\gamma P_o \int e^{i(x-ct)}\partial t$$

$$p = -ic\gamma P_o \frac{e^{i(x-ct)}}{-ic} + f(x)$$

$$p = \gamma P_o e^{i(x-ct)} + f(x)$$

The function $f(x)$ is a constant of integration (That is constant with respect to time.)