# Homework #4

$$\int x \sin ax \, dx$$

Integration by parts

$u = x$

$dv = \sin ax\, dx$

$v = -\dfrac{\cos ax}{a}$

$du = dx$

$$\int u\, dv = uv - \int v\, du$$

$$\int x \sin ax\, dx = -x\frac{\cos ax}{a} - \int -\frac{\cos ax}{a}\, dx$$

$$-x\frac{\cos ax}{a} + \int \frac{\cos ax}{a}\, dx$$

$$\boxed{\int x \sin ax\, dx = -x\frac{\cos ax}{a} + \frac{\sin ax}{a^2}}$$

Now

$$A_n = \frac{2}{L}\int_0^L f(x)\sin\left(\frac{n\pi}{L}x\right)dx \qquad l = 1$$

$$A_n = 2\left[\int_0^{\frac{1}{4}} 4x \sin(n\pi x)\,dx + \int_{1/4}^{3/4} \sin(n\pi x)\,dx + \int_{3/4}^{1} (-4x+4)\sin(n\pi\right.$$

$$A_n = 8\int_0^{1/4} x\sin(n\pi x)\,dx + 2\int_{1/4}^{3/4}\sin(n\pi x\,dx) - 8\int_{3/4}^{1} x\sin(n\pi x)\,dx$$

$$+ 8\int_{3/4}^{1}\sin(n\pi x)\,dx$$

$$A_n = 8\left(-\frac{x\cos(n\pi x)}{n\pi} + \frac{\sin(n\pi x)}{(n\pi)^2}\right)\Bigg|_0^{1/4} + 2\left(-\frac{\cos(n\pi x)}{n\pi}\right)\Bigg|_{1/4}^{3/4}$$

$$-8\left(-\frac{x\cos(n\pi x)}{n\pi} + \frac{\sin(n\pi x)}{(n\pi)^2}\right)\Bigg|_{3/4}^{1} + 8\left(-\frac{\cos(n\pi x)}{n\pi}\right)\Bigg|_{3/4}^{1}$$

$$A_n = 8\left(-\frac{\cos(n\pi/4)}{4n\pi} + \frac{\sin(n\pi/4)}{(n\pi)^2} - 0 - 0\right)$$
$$-2\left(\frac{\cos(3n\pi/4)}{n\pi} - \frac{\cos(n\pi/4)}{n\pi}\right)$$
$$-8\left(\frac{-\cos(n\pi)}{n\pi} + \frac{\sin(n\pi)}{(n\pi)^2} + \frac{3\cos(3n\pi/4)}{4n\pi} - \frac{\sin(3n\pi/4)}{(n\pi)^2}\right)$$
$$-8\left(\frac{\cos(n\pi)}{n\pi} - \frac{\cos(3n\pi/4)}{n\pi}\right)$$

Now $\sin(n\pi) = 0$ + $\cos(n\pi) = (-1)^n$

$$A_n = -\frac{2\cos(n\pi/4)}{n\pi} + \frac{8\sin(n\pi/4)}{(n\pi)^2} - \frac{2\cos(3n\pi/4)}{n\pi} + \frac{2\cos(n}{n\pi}$$
$$\cancel{+8\cos} + \frac{8(-1)^n}{n\pi} - \frac{8\cdot 0}{(n\pi)^2} - \frac{6\cos(3n\pi/4)}{n\pi} + \frac{8\sin(3n\pi/}{(n\pi)^2}$$
$$-\frac{8(-1)^n}{n\pi} + \frac{8\cos(3n\pi/4)}{n\pi}$$

1st + 4th terms cancel
5th + 9th terms cancel
3rd, 7th + 10th terms cancel

$$A_n = \frac{8}{(n\pi)^2}\left[\sin(n\pi/4) + \sin(3n\pi/4)\right]$$

$$A_1 = \frac{8}{(1\pi)^2}\left[\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\right] = \frac{8\sqrt{2}}{\pi^2} = 1.146$$
$$A_2 = \frac{8}{(2\pi)^2}\left[\sin(\pi/2) + \sin(3\pi/2)\right] = 0$$
$$A_3 = \frac{8}{(3\pi)^2}\left[\sin(3\pi/4) + \sin(9\pi/4)\right] = \frac{8}{9\pi^2}\left[\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\right] = \frac{8\sqrt{}}{9\pi}$$
$$A_3 = 0.127$$
$$A_4 = \frac{8}{(4\pi)^2}\left[\sin(\pi) + \sin(3\pi)\right] = 0$$

$$A_5 = \frac{8}{(5\pi)^2}\left[\sin(5\pi/4) + \sin(15\pi/4)\right] = \frac{8}{25\pi^2}\left[-\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}\right]$$

$$A_5 = \frac{-8\sqrt{2}}{25\pi^2} = -0.046$$

frequencies $f_n = \frac{nc}{2L}$ $\quad L = 1 \quad c = 100$

$$\therefore f_n = \frac{n\,100}{2\cdot 1} = 50n$$

| Harmonic | Frequency (Hz) | Amplitude |
|---|---|---|
| 1 | 50 | 1.146 |
| 2 | 100 | 0 |
| 3 | 150 | 0.127 |
| 4 | 200 | 0 |
| 5 | 250 | -0.046 |