Chapter 38 Problem 71 †

Given
$4p \rightarrow He + 27\ MeV$
$m_{Sun} = 2.0 \times 10^{30}\ kg$

Solution
a) At what rate does the Sun consume protons to produce a power of $4 \times 10^{26}\ W$?

This is basically a unit conversion problem. We know the number of protons to generate a certain amount of energy. The energy is given in electron-volts. This needs to be converted into joules. Also remember that a watt is the same as joules per second. Therefore,

$$P = 4 \times 10^{26}\ \frac{J}{s} \left(\frac{1\ eV}{1.6 \times 10^{-19}\ J}\right) \left(\frac{4\ p}{27 \times 10^{6}\ eV}\right) = 3.70 \times 10^{38}\ p/s$$

b) Find the length of the Sun's present phase if the original amount of hydrogen is 71% and the phase ends when 10% of the hydrogen is consumed.

First find the original mass of hydrogen and then take 10% of that value. This gives

$$m_H = (0.71)(2.0 \times 10^{30}\ kg) = 1.42 \times 10^{30}\ kg$$

Hydrogen consumed by the end of the present phase is

$$\Delta H = (0.10)(1.42 \times 10^{30}\ kg) = 1.42 \times 10^{29}\ kg$$

Now convert this value into the number of hydrogen atoms, which is also the number of protons involved in the fusion reaction.

$$\Delta H = 1.42 \times 10^{29}\ kg \left(\frac{1\ p}{1.67 \times 10^{-27}\ kg}\right) = 8.50 \times 10^{55}\ p$$

Since power is energy per time, then

$$P = \frac{E}{t}$$

$$t = \frac{E}{P}$$

From the calculations done above, the energy is in terms of protons and the power is in terms of protons per second. Then the time is

$$t = \frac{8.50 \times 10^{55}\ p}{3.70 \times 10^{38}\ p/s} = 2.30 \times 10^{17}\ s$$

This comes out to $7.3 \times 10^{9}\ yrs$ or 7.3 billion years.

† Problem from Essential University Physics, Wolfson