Chapter 37 Problem 32 †

Solution

Show the formula for calculating the wavelength of an emitted photon for the transition from l to $l-1$ state where the rotational inertia is I.

The energy of rotation for the state l is

$$E_l = \frac{\hbar^2}{2I} l(l+1)$$

The energy of rotation for the state $l-1$ is

$$E_{l-1} = \frac{\hbar^2}{2I}(l-1)((l-1)+1) = \frac{\hbar^2}{2I}(l-1)l$$

The difference in energy is then

$$\Delta E = E_l - E_{l-1} = \frac{\hbar^2}{2I} l(l+1) - \frac{\hbar^2}{2I}(l-1)l$$

$$\Delta E = \frac{\hbar^2}{2I}\left(l(l+1) - (l-1)l\right) = \frac{\hbar^2}{2I}\left(l^2 + l - l^2 + l\right) = \frac{\hbar^2}{2I} 2l = \frac{\hbar^2 l}{I}$$

Replacing $\hbar$ with $h/2\pi$ gives

$$\Delta E = \frac{h^2 l}{4\pi^2 I}$$

The relationship between energy and wavelength of a photon is

$$E = \frac{hc}{\lambda}$$

Set these equal to each other and solve for wavelength gives

$$\frac{h^2 l}{4\pi^2 I} = \frac{hc}{\lambda}$$

$$\lambda = \frac{hc4\pi^2 I}{h^2 l}$$

$$\lambda = \frac{4\pi^2 I c}{hl}$$

†Problem from Essential University Physics, Wolfson