Chapter 36 Problem 45 [†]

Given
$State = 4F_{5/2}$

Solution
a) Find its energy in units of ground-state energy.

The principle quantum number is tied to the energy of the electron. For the hydrogen atom this is given by the formula

$$E_n = \frac{-13.6\ eV}{n^2}$$

Since the ground state is $n = 1$, then the energy in units of ground-state energy is

$$E_n = \frac{E_1}{n^2}$$

The first number of the quantum state indicates that $n = 4$, so the energy is

$$E = \frac{E_1}{16}$$

b) Find the orbital angular momentum in units of $\hbar$.

Angular momentum is given by the formula

$$L = \sqrt{l(l+1)}\hbar$$

From the quantum state of the orbital angular momentum, F corresponds to $l = 3$. Therefore,

$$L = \sqrt{3(3+1)}\hbar = \sqrt{12}\hbar$$

In units of $\hbar$, the orbital angular momentum is

$$L = \sqrt{12}$$

c) Find the magnitude of the total angular momentum in units of $\hbar$.

The total angular momentum is given by the formula

$$J = \sqrt{j(j+1)}\hbar$$

The quantum state of the total angular momentum is $j = 5/2$. Therefore,

$$J = \sqrt{(5/2)(5/2+1)}\hbar = \sqrt{(5/2)(7/2)}\hbar = \sqrt{35/4}\hbar = \tfrac{1}{2}\sqrt{35}\hbar$$

In units of $\hbar$ the total angular momentum is

$$J = \tfrac{1}{2}\sqrt{35}$$

[†]Problem from Essential University Physics, Wolfson