Chapter 35 Problem 47 †

Solution

Find the probability of finding a particle in the ground state of an infinite square well with a detector that has a resolution of 15% of the well width.

The normalized wave function for a particle in the ground state of an infinite square well is

$$\psi = \sqrt{\frac{2}{L}} \sin\left(\frac{\pi}{L}x\right)$$

To find the probability integrate the square of the wave function over the interval of interest.

$$P(x) = \psi^2 dx$$

$$P = \int_{x_0}^{x_1} \left(\sqrt{\frac{2}{L}} \sin\left(\frac{\pi}{L}x\right)\right)^2 dx$$

$$P = \int_{x_0}^{x_1} \frac{2}{L} \sin^2\left(\frac{\pi}{L}x\right) dx$$

$$P = \frac{2}{L} \int_{x_0}^{x_1} \frac{1}{2}\left(1 - \cos\left(\frac{2\pi}{L}x\right)\right) dx$$

$$P = \frac{1}{L}\left(x - \frac{\sin\left(\frac{2\pi}{L}x\right)}{2\pi/L}\right\vert_{x_0}^{x_1}$$

$$P = \frac{1}{L}\left(x_1 - \frac{L}{2\pi}\sin\left(\frac{2\pi}{L}x_1\right) - x_0 + \frac{L}{2\pi}\sin\left(\frac{2\pi}{L}x_0\right)\right) \tag{1}$$

a) Find the probability centered on the middle of the well.

Since the detector has a resolution of $0.15L$, then half of this range is on each side of $0.5L$ gives

$$x_0 = 0.50L - \tfrac{1}{2}0.15L = 0.425L$$

$$x_1 = 0.50L + \tfrac{1}{2}0.15L = 0.575L$$

From equation (1) the probability is

$$P = \frac{1}{L}\left(0.575L - \frac{L}{2\pi}\sin\left(\frac{2\pi}{L}0.575L\right) - 0.425L + \frac{L}{2\pi}\sin\left(\frac{2\pi}{L}0.425L\right)\right)$$

$$P = \frac{1}{L}\left(0.150L - \frac{L}{2\pi}\sin\left(2\pi\ 0.575\right) + \frac{L}{2\pi}\sin\left(2\pi\ 0.425\right)\right)$$

$$P = \frac{1}{L}\left(0.150L - \frac{L}{2\pi}\sin\left(1.15\pi\right) + \frac{L}{2\pi}\sin\left(0.850\pi\right)\right)$$

$$P = \frac{1}{L}\left(0.150L - \frac{L}{2\pi}(-0.454) + \frac{L}{2\pi}(0.454)\right) = 0.295$$

b) Find the probability centered on the first quarter of the well.

†Problem from Essential University Physics, Wolfson

Since the detector has a resolution of $0.15L$, then half of this range is on each side of $0.25L$ gives

$$x_0 = 0.25L - \tfrac{1}{2}0.15L = 0.175L$$

$$x_1 = 0.25L + \tfrac{1}{2}0.15L = 0.325L$$

From equation (1) the probability is

$$P = \frac{1}{L}\left(0.325L - \frac{L}{2\pi}\sin\left(\frac{2\pi}{L}0.325L\right) - 0.175L + \frac{L}{2\pi}\sin\left(\frac{2\pi}{L}0.175L\right)\right)$$

$$P = \frac{1}{L}\left(0.150L - \frac{L}{2\pi}\sin\left(2\pi\ 0.325\right) + \frac{L}{2\pi}\sin\left(2\pi\ 0.175\right)\right)$$

$$P = \frac{1}{L}\left(0.150L - \frac{L}{2\pi}\sin\left(0.650\pi\right) + \frac{L}{2\pi}\sin\left(0.350\pi\right)\right)$$

$$P = \frac{1}{L}\left(0.150L - \frac{L}{2\pi}(0.891) + \frac{L}{2\pi}(0.891)\right) = 0.15$$