Chapter 33 Problem 60 †

Given

$1\ eV = 1.6 \times 10^{-19}\ J$
$c = 3.00 \times 10^{8}\ m/s$
$m_e = 9.11 \times 10^{-31}\ kg$

Solution

a) Find the speed of an electron with a kinetic energy of 100 eV.

The relationship between kinetic energy and velocity is

$$E = \gamma mc^2 = K + mc^2$$

$$\frac{1}{\sqrt{1-(v/c)^2}} mc^2 = K + mc^2$$

where m is the rest mass of the electron and K is the kinetic energy of the electron. Solving for velocity gives

$$\frac{1}{\sqrt{1-(v/c)^2}} = \frac{K}{mc^2} + 1$$

$$\frac{1}{1-(v/c)^2} = \left(\frac{K}{mc^2} + 1\right)^2$$

$$1-(v/c)^2 = \frac{1}{\left(\frac{K}{mc^2} + 1\right)^2}$$

$$(v/c)^2 = 1 - \frac{1}{\left(\frac{K}{mc^2} + 1\right)^2}$$

$$v = c\sqrt{1 - \frac{1}{\left(\frac{K}{mc^2} + 1\right)^2}} \tag{1}$$

Next convert the kinetic energy into joules and substitute the appropriate values into equation 1.

$$K = (100\ eV)\left(\frac{1.6 \times 10^{-19}\ J}{1\ eV}\right) = 1.6 \times 10^{-17}\ J$$

$$v = c\sqrt{1 - \frac{1}{\left(\frac{1.6\times10^{-17}\ J}{(9.11\times10^{-31}\ kg)(3.00\times10^{8}\ m/s)^2} + 1\right)^2}} = 0.0198c$$

b) Find the speed of an electron with a kinetic energy of 100 keV.

Convert the kinetic energy into joules and substitute the appropriate values into equation 1.

$$K = (100 \times 10^{3}\ eV)\left(\frac{1.6 \times 10^{-19}\ J}{1\ eV}\right) = 1.6 \times 10^{-14}\ J$$

$$v = c\sqrt{1 - \frac{1}{\left(\frac{1.6\times10^{-14}\ J}{(9.11\times10^{-31}\ kg)(3.00\times10^{8}\ m/s)^2} + 1\right)^2}} = 0.548c$$

†Problem from Essential University Physics, Wolfson

c) Find the speed of an electron with a kinetic energy of 1.0 MeV.

Convert the kinetic energy into joules and substitute the appropriate values into equation 1.

$$K = (1.0 \times 10^{6}\ eV)\left(\frac{1.6 \times 10^{-19}\ J}{1\ eV}\right) = 1.6 \times 10^{-13}\ J$$

$$v = c\sqrt{1 - \frac{1}{\left(\frac{1.6\times10^{-13}\ J}{(9.11\times10^{-31}\ kg)(3.00\times10^{8}\ m/s)^2} + 1\right)^2}} = 0.941c$$

d) Find the speed of an electron with a kinetic energy of 1.0 GeV.

Convert the kinetic energy into joules and substitute the appropriate values into equation 1.

$$K = (1.0 \times 10^{9}\ eV)\left(\frac{1.6 \times 10^{-19}\ J}{1\ eV}\right) = 1.6 \times 10^{-10}\ J$$

$$v = c\sqrt{1 - \frac{1}{\left(\frac{1.6\times10^{-10}\ J}{(9.11\times10^{-31}\ kg)(3.00\times10^{8}\ m/s)^2} + 1\right)^2}} = 0.99999987c$$