Chapter 33 Problem 54 †

Solution

Find the original velocity if the momentum triples when the velocity doubles.

The relativistic momentum of a particle is given by the equation

$$p = \gamma m v = \frac{mv}{\sqrt{1 - (v/c)^2}}$$

Since the momentum triples, then

$$p = 3p_0$$

$$\frac{mv}{\sqrt{1-(v/c)^2}} = 3\left(\frac{mv_0}{\sqrt{1-(v_0/c)^2}}\right)$$

But we know the velocity doubles, so $v = 2v_0$.

$$\frac{m(2v_0)}{\sqrt{1-(2v_0/c)^2}} = \frac{3mv_0}{\sqrt{1-(v_0/c)^2}}$$

$$\frac{2mv_0}{\sqrt{1-4(v_0/c)^2}} = \frac{3mv_0}{\sqrt{1-(v_0/c)^2}}$$

Multiply both sides by $\sqrt{1-4(v_0/c)^2}/(3mv_0)$ gives

$$\frac{2mv_0}{3mv_0} = \frac{\sqrt{1-4(v_0/c)^2}}{\sqrt{1-(v_0/c)^2}}$$

Simplfy and square both sides

$$\left(\frac{2}{3}\right)^2 = \frac{1-4(v_0/c)^2}{1-(v_0/c)^2}$$

Multiply both sides by $1-(v_0/c)^2$ and solve for v_0/c.

$$\tfrac{4}{9}(1-(v_0/c)^2) = 1-4(v_0/c)^2$$

$$\tfrac{4}{9} - \tfrac{4}{9}(v_0/c)^2 = 1-4(v_0/c)^2$$

$$4(v_0/c)^2 - \tfrac{4}{9}(v_0/c)^2 = 1 - \tfrac{4}{9}$$

Get a common denominator.

$$\tfrac{36}{9}(v_0/c)^2 - \tfrac{4}{9}(v_0/c)^2 = \tfrac{9}{9} - \tfrac{4}{9}$$

$$\tfrac{32}{9}(v_0/c)^2 = \tfrac{5}{9}$$

$$(v_0/c)^2 = \tfrac{5}{9}\tfrac{9}{32} = \tfrac{5}{32}$$

Therefore,

$$v_0 = c\sqrt{\tfrac{5}{32}} = 0.395c$$

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†Problem from Essential University Physics, Wolfson