Chapter 17 Problem 70 †

Given
$L_0 = 1.35\ m$
$T_0 = 20°C$
$T_f = 17°C$
$\alpha_{brass} = 19 \times 10^{-6}\ K^{-1}$

Solution
Find the time at which the clock will be in error by 1 minute.

First find the new length of the pendulum.

$$L_f = L_0(1 + \alpha \Delta T) = (1.35\ m)(1 + (19 \times 10^{-6}\ K^{-1})(17°C - 20°C)$$

$$L_f = 1.34992305\ m$$

Now the time period for a simple pendulum is

$$time = 2\pi\sqrt{\frac{L}{g}}$$

At room temperature the time period of the pendulum is

$$time_0 = 2\pi\sqrt{\frac{1.35\ m}{9.8\ m/s^2}}$$

$$time_0 = 2.332027754\ s$$

Given the size of the difference, a lot more significant figures are maintained that usual. The time period at the current temperature is

$$time_f = 2\pi\sqrt{\frac{1.34992305\ m}{9.8\ m/s^2}}$$

$$time_f = 2.33196129\ s$$

Since the pendulum is shorter, the time period is also shorter. The fraction of error in the new pendulum is

$$\epsilon = \frac{time_0 - time_f}{time_0} = \frac{2.332027754\ s - 2.33196129\ s}{2.332027754\ s} = 2.85005 \times 10^{-5}$$

This is an error of 0.00285005

$$\epsilon = \frac{cumulative\ error}{total\ time}$$

Solving for *total time* gives

$$total\ time = \frac{cumulative\ error}{\epsilon} = \frac{60\ s}{2.85005 \times 10^{-5}} = 2.105 \times 10^6\ s$$

Converting this to hours gives 584.8 hours or 24.37 days.

† Problem from Essential University Physics, Wolfson