

## Chapter 38 Problem 47 †

### Solution

How old is the archaeological find?

The half-life of carbon-14 is 5730 *yr*. Since the carbon-14 content is 34% of current day values, then

$$\frac{N}{N_0} = 0.34$$

Using the half-life radioactive decay equation

$$N = N_0 2^{-t/t_{1/2}}$$

Solve for  $t$

$$\frac{N}{N_0} = 2^{-t/t_{1/2}}$$

Take the log base-2 of each side

$$\log_2 \left( \frac{N}{N_0} \right) = -t/t_{1/2}$$

$$t = -t_{1/2} \log_2 \left( \frac{N}{N_0} \right) = -(5730 \text{ yr}) \log_2 (0.34)$$

Therefore,

$$t = -(5730 \text{ yr})(-1.556) = 8916 \text{ yr}$$

Most of us do not have calculators that do  $\log_2$ . Therefore, the more reliable way of solving this problem is to find the decay constant first.

$$\lambda = \frac{\ln 2}{t_{1/2}} = \frac{\ln 2}{5730 \text{ yr}} = 1.210 \times 10^{-4} \text{ yr}^{-1}$$

Now use the base-e model

$$N = N_0 e^{-\lambda t}$$

Divide by  $N_0$  and take the natural log of both sides

$$\ln \left( \frac{N}{N_0} \right) = -\lambda t$$

Therefore,

$$t = \frac{-\ln \left( \frac{N}{N_0} \right)}{\lambda} = \frac{-\ln(0.34)}{1.210 \times 10^{-4} \text{ yr}^{-1}} = 8916 \text{ yr} = 8900 \text{ yr}$$

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†Problem from Essential University Physics, Wolfson