Chapter 35 Problem 29 [†]

Given

$\lambda = 950\ nm = 950 \times 10^{-9}\ m$

Solution

What are the dimensions of a cubical box with an electron in it?

The energy of the photon emitted in the transition to the ground state is

$$E = \frac{hc}{\lambda} = \frac{(6.63 \times 10^{-34}\ J \cdot s)(3.0 \times 10^{8}\ m/s)}{950 \times 10^{-9}\ m} = 2.09 \times 10^{-19}\ J$$

The energy states for a particle in a cubical box are

$$E_{n_x n_y n_z} = \frac{h^2}{8mL^2}\left(n_x^2 + n_y^2 + n_z^2\right)$$

The ground state energy is

$$E_{111} = \frac{h^2}{8mL^2}\left(1^2 + 1^2 + 1^2\right) = \frac{3h^2}{8mL^2}$$

The next excited state has an energy of

$$E_{211} = \frac{h^2}{8mL^2}\left(2^2 + 1^2 + 1^2\right) = \frac{6h^2}{8mL^2}$$

Therefore the energy of transition is

$$E = E_f - E_i = E_{111} - E_{211} = \frac{3h^2}{8mL^2} - \frac{6h^2}{8mL^2} = -\frac{3h^2}{8mL^2}$$

This energy loss is an energy gain by the photon. Dropping the negative sign and solving for L gives

$$L^2 = \frac{3h^2}{8mE}$$

$$L = \sqrt{\frac{3h^2}{8mE}}$$

Substituting in the appropriate values gives

$$L = \sqrt{\frac{3(6.63 \times 10^{-34}\ J \cdot s)^2}{8(9.11 \times 10^{-31}\ kg)(2.09 \times 10^{-19}\ J)}} = 9.3 \times 10^{-10}\ m$$

[†]Problem from Essential University Physics, Wolfson