Chapter 34 Problem 61 †

Given

$q = 2e$

Solution

a) Find the radius of the ground-state electron in a singly ionized Helium atom, He^-.

The radius of the ground-state can be found using the Bohr radius

$$a_0 = \frac{\hbar^2}{mke^2} = \frac{(h/2\pi)^2}{mke^2} = \frac{h^2}{4\pi^2 mke^2}$$

The value of e^2 in the denominator is due to the charge of the electron and the charge of the nucleus. Since the nucleus has twice the charge, we need to replace e^2 with $2e^2$.

$$a_0 = \frac{h^2}{4\pi^2 mk2e^2} = \frac{h^2}{8\pi^2 mke^2}$$

Substituting in the appropriate values gives

$$a_0 = \frac{(6.63 \times 10^{-34}\ J \cdot s)^2}{8\pi^2(9.11 \times 10^{-31}\ kg)(8.99 \times 10^9\ Nm^2/C^2)(1.6 \times 10^{-19}\ C)^2} = 2.66 \times 10^{-11}\ m = 26.6\ pm$$

b) Find the photon energy emitted when the single electron makes a transition from the $n = 2$ to the $n = 1$ state.

The energy level of each state is given by the equation

$$E = -\frac{ke^2}{2a_0}\left(\frac{1}{n^2}\right)$$

Since the nuclear charge is doubled, the value of e^2 in the numerator should go to $2e^2$ as discussed in part a. This gives

$$E = -\frac{k2e^2}{2a_0}\left(\frac{1}{n^2}\right) = -\frac{ke^2}{a_0}\left(\frac{1}{n^2}\right)$$

The energy for state $n = 1$ is

$$E_1 = -\frac{(8.99 \times 10^9\ Nm^2/C^2)(1.6 \times 10^{-19}\ C)^2}{2.66 \times 10^{-11}\ m}\left(\frac{1}{1^2}\right) = -8.65 \times 10^{-18}\ J$$

The energy for state $n = 2$ is

$$E_2 = -\frac{(8.99 \times 10^9\ Nm^2/C^2)(1.6 \times 10^{-19}\ C)^2}{2.66 \times 10^{-11}\ m}\left(\frac{1}{2^2}\right) = -2.16 \times 10^{-18}\ J$$

The change in energy is then

$$\Delta E = E_1 - E_2 = -8.65 \times 10^{-18}\ J - (-2.16 \times 10^{-18}\ J) = -6.49 \times 10^{-18}\ J$$

A energy loss by the atom must be transmitted away as a photon. Therefore, the energy of the photon is $6.49 \times 10^{-18}\ J$. Converting this to electron volts gives us

$$\Delta E = (6.49 \times 10^{-18}\ J)\left(\frac{1\ eV}{1.6 \times 10^{-19}\ J}\right) = 40.6\ eV$$

†Problem from Essential University Physics, Wolfson