Chapter 19 Problem 56 †

Given

$m_{Cu} = 500\ g = 0.500\ kg$
$T_{Cu} = 80°C = 353\ K$
$m_{H2O} = 1.0\ kg$
$T_{H2O} = 10°C = 283\ K$
$c_{H2O} = 4184\ J/kg \cdot K$
$c_{Cu} = 386\ J/kg \cdot K$

Solution

a) Find the final temperature when the system reaches equilibrium.

Since the initial temperatures of the water and copper are between the melting point of ice and the boiling point of water, there will be no phase changes. Then the equation for keeping track of the heat transfer involves sensible heating/cooling.

$$\Delta Q_{Cu} + \Delta Q_{H2O} = 0$$

$$m_{Cu}c_{Cu}\Delta T_{Cu} + m_{H2O}c_{H2O}\Delta T_{H2O} = 0$$
$$m_{Cu}c_{Cu}(T - T_{Cu}) + m_{H2O}c_{H2O}(T - T_{H2O}) = 0$$

Solving for final temperature gives

$$m_{Cu}c_{Cu}T - m_{Cu}c_{Cu}T_{Cu} + m_{H2O}c_{H2O}T - m_{H2O}c_{H2O}T_{H2O} = 0$$

$$m_{Cu}c_{Cu}T + m_{H2O}c_{H2O}T = m_{Cu}c_{Cu}T_{Cu} + m_{H2O}c_{H2O}T_{H2O}$$
$$T = \frac{m_{Cu}c_{Cu}T_{Cu} + m_{H2O}c_{H2O}T_{H2O}}{m_{Cu}c_{Cu} + m_{H2O}c_{H2O}}$$
$$T = \frac{(0.500\ kg)(386\ J/kg \cdot K)(353\ K) + (1.0\ kg)(4184\ J/kg \cdot K)(283\ K)}{(0.500\ kg)(386\ J/kg \cdot K) + (1.0\ kg)(4184\ J/kg \cdot K)} = 286\ K$$

b) Find the entropy change of the system.

Since the temperature changes as the water/copper is warming/cooling we must integrate to get the entropy change.

$$\Delta S = \int_1^2 \frac{dQ}{T} \qquad (1)$$

The heat required to warm water is

$$\Delta Q = mc\Delta T$$

Therefore, for infinitesimal temperature changes

$$dQ = mcdT \qquad (2)$$

Substituting 2 into 1 and integrating gives

$$\Delta S = \int_{T_1}^{T_2} \frac{mcdT}{T} = mc \ln\left(\frac{T_2}{T_1}\right)$$

†Problem from Essential University Physics, Wolfson

For the water we get

$$\Delta S_{H2O} = (1.0\ kg)(4184\ J/kg \cdot K) \ln\left(\frac{286\ K}{283\ K}\right)$$

$$\Delta S_{H2O} = 44.1\ J/K$$

For the copper we get

$$\Delta S_{Cu} = (0.50\ kg)(386\ J/kg \cdot K) \ln\left(\frac{286\ K}{353\ K}\right)$$

$$\Delta S_{Cu} = -40.6\ J/K$$

The total change in entropy is then

$$\Delta S_{tot} = 44.1\ J/K - 40.6\ J/K = 3.5\ J/K$$