

Chapter 19 Problem 43 †

Given

$$m = 94 \text{ Mg} = 94,000 \text{ kg}$$

$$T_1 = 0^\circ\text{C} = 273 \text{ K}$$

$$T_2 = 15^\circ\text{C} = 288 \text{ K}$$

$$L_f = 334 \text{ kJ/kg}$$

Solution

Find the entropy change when melting ice and then heating the water.

The heat flow into the ice during melting is

$$\Delta Q = mL_f = (94,000 \text{ kg})(334 \text{ kJ/kg})$$

$$\Delta Q = 3.14 \times 10^{10} \text{ J}$$

The melting of the ice is at constant temperature. Therefore, the entropy change is

$$\Delta S = \frac{\Delta Q}{T} = \frac{3.14 \times 10^{10} \text{ J}}{273 \text{ K}} = 1.15 \times 10^8 \text{ J/K}$$

Since the temperature changes as the water is warming we must integrate to get the entropy change.

$$\Delta S = \int_1^2 \frac{dQ}{T} \tag{1}$$

The heat required to warm water is

$$\Delta Q = mc\Delta T$$

Therefore, for infinitesimal temperature changes

$$dQ = mcdT \tag{2}$$

Substituting 2 into 1 and integrating gives

$$\Delta S = \int_{T_1}^{T_2} \frac{mcdT}{T} = mc \ln \left(\frac{T_2}{T_1} \right)$$

$$\Delta S = (94,000 \text{ kg})(4184 \text{ J/kg} \cdot \text{K}) \ln \left(\frac{288 \text{ K}}{273 \text{ K}} \right)$$

$$\Delta S = 2.10 \times 10^7 \text{ J/K}$$

$$\Delta S = \Delta S_{\text{melt}} + \Delta S_{\text{warm}}$$

$$\Delta S_{\text{tot}} = 1.15 \times 10^8 \text{ J/K} + 2.10 \times 10^7 \text{ J/K} = 1.36 \times 10^8 \text{ J/K}$$

$$\Delta S_{\text{tot}} = 136 \text{ MJ/K}$$

†Problem from Essential University Physics, Wolfson