Chapter 18 Problem 62 †

Given
$p = 10v^3 - 67v^2 + 220v$
$v_0 = 0$
$v_f = 4.5\ mL$

Solution
Find the work done as the lungs inflate.

The definition of work is

$$W = -\int_{v_0}^{v_f} p dv$$

$$W = -\int_{0}^{4.5} (10v^3 - 67v^2 + 220v) dv$$

$$W = -\left(\frac{10}{4}v^4 - \frac{67}{3}v^3 + \frac{220}{2}v^2 \right|_0^{4.5}$$

$$W = -\left(\frac{10}{4}(4.5)^4 - \frac{67}{3}(4.5)^3 + \frac{220}{2}(4.5)^2 \right)$$

$$W = -(1025 - 2035 + 2228) = -1218$$

The negative work indicates that the environment did negative work on the lungs, which means the lungs did positive work on the environment.

We have to be careful about the units in this problem. The pressure is in pascals and the volume is in milliliters. Implied units could be determined to get the following result, but I will make an argument from the original definition of work. Since work is an integral of pressure with respect to volume, then the units are (Pa) X (mL). Therefore, the final answer is in the same set of units. To get work into joules, the volume needs to be converted to cubic meters. Since 1000 liters is in a cubic meter and 1000 milliliters in a liter, then there are 1,000,000 mL in m^3. Therefore, the answer is

$$W = -1218 \left(\frac{1\ m^3}{1 \times 10^6\ mL} \right) = -1218 \times 10^{-6}\ J = -1.22 mJ$$

†Problem from Essential University Physics, Wolfson