

Chapter 38 Problem 64 †

Given

$$4p \rightarrow He + 27 \text{ MeV}$$
$$m_{Sun} = 2.0 \times 10^{30} \text{ kg}$$

Solution

a) At what rate does the Sun consume protons to produce a power of $4 \times 10^{26} \text{ W}$?

This is basically a unit conversion problem. We know the number of protons to generate a certain amount of energy. The energy is given in electron-volts. This needs to be converted into joules. Also remember that a watt is the same as joules per second. Therefore,

$$P = 4 \times 10^{26} \frac{J}{s} \left(\frac{1 \text{ eV}}{1.6 \times 10^{-19} \text{ J}} \right) \left(\frac{4 p}{27 \times 10^6 \text{ eV}} \right) = 3.70 \times 10^{38} \text{ p/s}$$

b) Find the length of the Sun's present phase if the original amount of hydrogen is 71% and the phase ends when 10% of the hydrogen is consumed.

First find the original mass of hydrogen and then take 10% of that value. This gives

$$m_H = (0.71)(2.0 \times 10^{30} \text{ kg}) = 1.42 \times 10^{30} \text{ kg}$$

Hydrogen consumed by the end of the present phase is

$$\Delta H = (0.10)(1.42 \times 10^{30} \text{ kg}) = 1.42 \times 10^{29} \text{ kg}$$

Now convert this value into the number of hydrogen atoms, which is also the number of protons involved in the fusion reaction.

$$\Delta H = 1.42 \times 10^{29} \text{ kg} \left(\frac{1 p}{1.67 \times 10^{-27} \text{ kg}} \right) = 8.50 \times 10^{55} p$$

Since power is energy per time, then

$$P = \frac{E}{t}$$

$$t = \frac{E}{P}$$

From the calculations done above, the energy is in terms of protons and the power is in terms of protons per second. Then the time is

$$t = \frac{8.50 \times 10^{55} p}{3.70 \times 10^{38} \text{ p/s}} = 2.30 \times 10^{17} \text{ s}$$

This comes out to $7.3 \times 10^9 \text{ yrs}$ or 7.3 billion years.

†Problem from Essential University Physics, Wolfson