

Chapter 34 Problem 65 †

Given

$$\Delta x = 20 \text{ nm}$$

$$m_e = 9.11 \times 10^{-31} \text{ kg}$$

$$h = 6.63 \times 10^{-34} \text{ J} \cdot \text{s}$$

Solution

Find the minimum speed of an electron trapped in a quantum well of width 20 nm.

Begin with the Heisenberg Uncertainty Principle

$$\Delta x \Delta p \geq \frac{h}{2\pi}$$

If you use the distinction that the direction of the momentum is unknown, then $\Delta p = p - (-p) = 2p$. See example 34.6 in the textbook. Applying this to Heisenberg's Uncertainty Principles gives

$$\Delta x 2p \geq \frac{h}{2\pi}$$

$$\Delta x p \geq \frac{h}{4\pi}$$

Often when Heisenberg's Uncertainty Principle is expressed, the distinction between left and right travelling particles is taken into account and the equation is

$$\Delta x \Delta p \geq \frac{h}{4\pi}$$

I will use the later form. Now momentum is mv . Therefore, the minimum velocity can not be known to be less than the uncertainty of the velocity. Therefore,

$$\Delta x m \Delta v = \frac{h}{4\pi}$$

$$\Delta v = \frac{h}{4\pi m \Delta x}$$

Substituting in the provided values gives

$$\Delta v = \frac{6.63 \times 10^{-34} \text{ J} \cdot \text{s}}{4\pi(9.11 \times 10^{-31} \text{ kg})(20 \times 10^{-9} \text{ m})} = 2900 \text{ m/s}$$

†Problem from Essential University Physics, Wolfson