Chapter 34 Problem 52 †

Given
$\lambda = 0.10\ nm = 1.0 \times 10^{-10}\ m$
$\theta = 90°$

Solution
What will be the final kinetic energy of an electron which was originally at rest?

We are dealing with Compton scattering and need to find the final energy of the photon. First find the change in wavelength of the photon as it scatters off an electron.

$$\Delta\lambda = \frac{h}{mc}(1 - \cos\theta)$$

$$\Delta\lambda = \frac{6.63 \times 10^{-34}\ J \cdot s}{(9.11 \times 10{-}31\ kg)(3.0 \times 10^{8}\ m/s)}(1 - \cos(90°)) = 2.43 \times 10^{-12}\ m$$

So the final wavelength of the photon is

$$\lambda_f = \lambda_o + \Delta\lambda = 1.0 \times 10^{-10}\ m + 2.43 \times 10^{-12}\ m = 1.0243 \times 10^{-10}\ m$$

The energy of the incoming photon is

$$E_o = hf = \frac{hc}{\lambda_0} = \frac{(6.63 \times 10^{-34}\ J \cdot s)(3.0 \times 10^{8}\ m/s)}{1.0 \times 10^{-10}\ m} = 1.989 \times 10^{-15}\ J$$

The energy of the outgoing photon is

$$E_f = hf = \frac{hc}{\lambda_0} = \frac{(6.63 \times 10^{-34}\ J \cdot s)(3.0 \times 10^{8}\ m/s)}{1.0243 \times 10^{-10}\ m} = 1.942 \times 10^{-15}\ J$$

The difference in energy is

$$\Delta E = E_f - E_0 = 1.942 \times 10^{-15}\ J - 1.989 \times 10^{-15}\ J = -0.047 \times 10^{-15}\ J = -4.7 \times 10^{-17}\ J$$

A loss of energy by the photon is a gain by the electron. Therefore, the electron has a final energy of $4.7 \times 10^{-17}\ J$. Converting this to electron-volts gives

$$\Delta E = 4.7 \times 10^{-17}\ J \left(\frac{1\ eV}{1.6 \times 10^{-19}\ J}\right) = 294\ eV$$

† Problem from Essential University Physics, Wolfson