Chapter 17 Problem 71 †

Given

$L_0 = 20.0\ cm = 0.20\ m$
$T_0 = 20°C$
$T_f = 18°C$
$\alpha_{brass} = 19 \times 10^{-6}\ K^{-1}$

Solution

Find the time at which the clock will be in error by 1 minute.

First find the new length of the pendulum.

$$L_f = L_0(1 + \alpha\Delta T) = (0.20\ m)(1 + (19 \times 10^{-6}\ K^{-1})(18°C - 20°C)$$

$$L_f = 0.1999924\ m$$

Now the time period for a simple pendulum is

$$time = 2\pi\sqrt{\frac{L}{g}}$$

At room temperature the time period of the pendulum is

$$time_0 = 2\pi\sqrt{\frac{0.2000\ m}{9.8\ m/s^2}}$$

$$time_0 = 0.8975979\ s$$

Given the size of the difference, a lot more significant figures are maintained that usual. The time period at the current temperature is

$$time_f = 2\pi\sqrt{\frac{0.1999924\ m}{9.8\ m/s^2}}$$

$$time_f = 0.8975808\ s$$

Since the pendulum is shorter, the time period is also shorter. The fraction of error in the new pendulum is

$$\epsilon = \frac{time_f - time_0}{time_0} = \frac{0.8975979\ s - 0.8975808\ s}{0.8975979\ s} = 1.905 \times 10^{-5}$$

This is an error of 0.001905

$$\epsilon = \frac{cumulative\ error}{total\ time}$$

Solving for *total time* gives

$$total\ time = \frac{cumulative\ error}{\epsilon} = \frac{60\ s}{1.905 \times 10^{-5}} = 3.1496 \times 10^{6}\ s$$

Converting this to hours gives 874.9 hours or 36.45 days.

†Problem from Essential University Physics, Wolfson