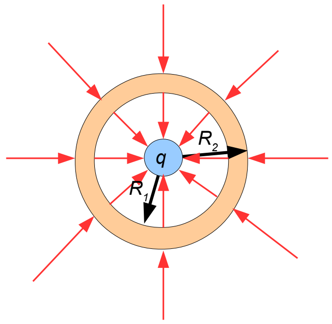
Chapter 7 Problem 98 [†]



Given
$R_1 = 2.0\ cm = 0.020\ m$
$R_2 = 2.2\ cm = 0.022\ m$
$q = -10\mu C = -1.0 \times 10^{-5}\ C$
$k = 8.99 \times 10^9\ Nm^2/C^2$

Solution
a) Find the electric potential of the gold shell.

The gold shell had no charge on it. When the silver coated pith ball is placed inside the shell, positive charge equal in magnitude to the charge on the pith ball migrates to the inner surface of the gold shell. As a result, there is no electric field inside the conductor. As positive charge moves to the inner surface, an equal negative charge accumulates on the outer surface of the gold shell. For anybody outside the gold shell (Gaussian surface), they see the original charge on the pith ball.
Now that we know that the charge on the surface of the gold shell is equal to q, we can calculate the voltage of the gold shell. It is spherically symmetric, so

$$V = \frac{kq}{R_2} = \frac{(8.99 \times 10^9\ Nm^2/C^2)(-1.0 \times 10^{-5}\ C)}{0.022\ m}$$

$$V = -4.09 \times 10^6\ V$$

Since there is no electric field inside the conductor, the entire gold shell is at this potential.

b) How much charge should be placed on the gold shell to make the potential equal to 100 V.

If charge is added to the gold surface, then the total charge of the system is

$$Q_{tot} = q + q_{shell} \qquad (Eq.1)$$

Because of spherical symmetry, the relationship for determining the voltage is again

$$V = \frac{kQ_{tot}}{R_2}$$

Solving for Q_{tot} gives

$$Q_{tot} = \frac{VR_2}{k} = \frac{(100\ V)(0.022\ m)}{8.99 \times 10^9\ Nm^2/C^2} = 2.44 \times 10^{-10}\ C$$

[†]Problem from Univesity Physics by Ling, Sanny and Moebs (OpenStax)

Use equation 1) to solve the charge on the shell.

$$q_{shell} = Q_{tot} - q$$

$$q_{shell} = (2.44 \times 10^{-10}\ C) - (-1.0 \times 10^{-5}\ C)$$

$$q_{shell} = +1.0000244 \times 10^{-5}\ C$$

This answer is beyond the number of significant digits of the problem. We have to assume the given charge on the pith ball is good to 8 significant digits.