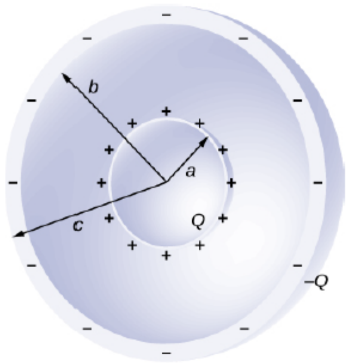
Chapter 7 Problem 65 [†]



Solution
Find the potential difference between the shells.

Since the inner shell has $+Q$ charge and the outer shell has $-Q$ charge, all the electric field in the system is located between the two shells. By Gauss's law for a spherical surface inside the inner shell, there is no enclosed charge; therefore, no electric field. Likewise, for a spherical shell outside the outer shell, it will encompass $(+Q) + (-Q) = 0$ charge. Therefore, there is no electric field here either.
Since the shells are spherically symmetry, the electric field between them is

$$\vec{E} = \frac{kq_{enc}}{r^2}\hat{r}$$

Notice, the enclosed charge is $+Q$. The voltage between the shells is then the integral going from $r_0 = a$ to $r_f = b$.

$$\Delta V = -\int \vec{E} \cdot d\vec{r} = -\int_a^b \left(\frac{kQ}{r^2}\hat{r}\right) \cdot dr\hat{r}$$

$$\Delta V = -\int_a^b \frac{kQ}{r^2}dr = -kQ\int_a^b \frac{1}{r^2}dr$$

$$\Delta V = -kQ\left(\frac{-1}{r}\right|_a^b$$

$$\Delta V = kQ\left(\frac{1}{b} - \frac{1}{a}\right)$$

The way the integral was set up, we are going from $r = a$ to $r = b$. This is in the same direction as the electic field. Therefore, the potential difference will be negative. Since $b > a$, this is consistent with what we would expect. If we want a positive potential, we should swap the limits and get

$$\Delta V = kQ\left(\frac{1}{a} - \frac{1}{b}\right)$$

[†]Problem from Univesity Physics by Ling, Sanny and Moebs (OpenStax)