

Chapter 7 Problem 56 †

Given

$$V = -xy^2z + 4xy$$

Solution

Find the electric field in this region.

Going from electric potential to electric field, you need to find the negative gradient of the potential given by the relationship

$$\vec{E} = -\nabla V$$

In Cartesian coordinate the operator expands out to

$$\vec{E} = -\frac{\partial V}{\partial x}\hat{i} - \frac{\partial V}{\partial y}\hat{j} - \frac{\partial V}{\partial z}\hat{k}$$

The partial derivative symbol means take an explicit derivative of the variable. Treat the other variables in the expression as a constant. This means you don't have to perform the chain rule to complete the derivative. Substitute in our expression and take the appropriate partials.

$$\vec{E} = -\frac{\partial(-xy^2z + 4xy)}{\partial x}\hat{i} - \frac{\partial(-xy^2z + 4xy)}{\partial y}\hat{j} - \frac{\partial(-xy^2z + 4xy)}{\partial z}\hat{k}$$

$$\vec{E} = -(-y^2z + 4y)\hat{i} - (-2xyz + 4x)\hat{j} - (-xy^2)\hat{k}$$

$$\vec{E} = (y^2z - 4y)\hat{i} + (2xyz - 4x)\hat{j} + (xy^2)\hat{k}$$

†Problem from University Physics by Ling, Sanny and Moebs (OpenStax)