Chapter 7 Problem 31 †

Given

$q_p = 1.60 \times 10^{-19}\ C$
$q_e = -1.60 \times 10^{-19}\ C$
$r = 0.529 \times 10^{-10}\ m = 5.29 \times 10^{-11}\ m$
$k = 8.99 \times 10^{9}\ Nm^2/C^2$

Solution

Find the work done as an electron moves from infinity to the average distance between a proton and electron in a hydrogen atom.

Since the electric field of the proton is spherically symmetry, it equals

$$\vec{E} = \frac{kq_p}{r^2}\hat{r}$$

The force on the electron is then

$$\vec{F} = \frac{kq_pq_e}{r^2}\hat{r}$$

The work done is then the integral as the elecctron moves from infinity to the distance r.

$$W = \int \vec{F} \cdot d\vec{r} = \int_{\infty}^{r} \left(\frac{kq_pq_e}{r^2}\hat{r}\right) \cdot dr\hat{r}$$

$$W = \int_{\infty}^{r} \frac{kq_pq_e}{r^2} dr = kq_pq_e \int_{\infty}^{r} \frac{1}{r^2} dr$$

$$W = kq_pq_e \left(\frac{-1}{r}\Big|_{\infty}^{r}\right.$$

$$W = kq_pq_e \left(\frac{-1}{r} - \frac{-1}{\infty}\right) = \frac{-kq_pq_e}{r}$$

Now substitute in the appropriate values gives

$$W = \frac{-(8.99 \times 10^{9}\ Nm^2/C^2)(1.60 \times 10^{-19}\ C)(-1.60 \times 10^{-19}\ C)}{5.29 \times 10^{-11}\ m}$$

$$W = 4.36 \times 10^{-18}\ J$$

Notice the work is positive. The electric force is directed towards the proton, attraction, and the electron is moving in the same direction. A negative sign appeared in the integral, but is cancelled by the negative charge of the electron. This amount of energy is small, so let's convert it to electron-volts.

$$W = (4.35 \times 10^{-18}\ J)\left(\frac{1\ eV}{1.6 \times 10^{-19}\ J}\right) = 27.2\ eV$$

Since the electron is in 'orbit' around the proton (an antiquated motion, but useful), half of the energy from this work is kinetic energy and half is potential energy. As a result, the potential energy of the electron bound to the proton is 13.6 eV, which is the ionization energy of the hydrogen atom.

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†Problem from Univesity Physics by Ling, Sanny and Moebs (OpenStax)