

## Chapter 7 Problem 104 †

### Given

$$\begin{aligned}r &= 0.529 \times 10^{-10} \text{ m} \\q_p &= 1.60 \times 10^{-19} \text{ C} \\q_e &= -1.60 \times 10^{-19} \text{ C} \\m_e &= 9.11 \times 10^{-31} \text{ kg} \\k &= 8.99 \times 10^9 \text{ Nm}^2/\text{C}^2\end{aligned}$$

### Solution

Find the angular velocity of the electron orbiting the nucleus of a hydrogen atom.

This question is treating the electron as if it were a planet orbiting the sun. This is not really what is happening because of the dual particle-wave nature of the electron. However, this simpler model is still consistent with the more complete quantum mechanical model. In this case the radius of the electron's orbit is equal to the most probable location for finding the electron. This is not the same as the average distance the electron is from the proton, which is a larger value. (Average distance is calculated by calculating a weighted average of distance and probability. I know this is more than what you wanted to know.)

Now let's do the problem. First the electric force is

$$F = \frac{kq_pq_e}{r^2} \quad (\text{Eq.1})$$

This force will be towards the center of the atom because the proton attracts the electron. Next, this force is equal to mass times centripetal acceleration.

$$F = ma = -m\frac{v^2}{r} \quad (\text{Eq.2})$$

The negative sign occurs because centripetal acceleration is towards the center of the circle.

We can combine these formulas to find the velocity of the electron, but we want angular velocity.

Remember that angular velocity is related to linear velocity by the following relationship.

$$v = \omega r \quad (\text{Eq.3})$$

Substitute this into equation (2) gives

$$F = -m\frac{(\omega r)^2}{r} = -m\omega^2 r$$

Set this equal to equation (1)

$$\frac{kq_pq_e}{r^2} = -m\omega^2 r$$

Don't let the negative sign cause a problem for you. It will be cancelled out when we substitute in the value of the electron's charge, which is also negative.

Now simplify and solve for  $\omega$ .

$$-\frac{kq_pq_e}{mr^3} = \omega^2$$

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†Problem from University Physics by Ling, Sanny and Moebs (OpenStax)

$$\omega = \sqrt{\frac{-kq_pq_e}{mr^3}}$$

Substituting in the appropriate values give

$$\omega = \sqrt{\frac{-(8.99 \times 10^9 \text{ Nm}^2/\text{C}^2)(1.60 \times 10^{-19} \text{ C})(-1.60 \times 10^{-19} \text{ C})}{(9.11 \times 10^{-31} \text{ kg})(0.529 \times 10^{-10})^3}}$$

$$\omega = \sqrt{1.71 \times 10^{33} \text{ rad}^2/\text{s}^2} = 4.13 \times 10^{16} \text{ rad/s}$$

This seems like a huge number, but calculating the velocity of the electron will give some context. Substitute this value into equation (3)

$$v = (4.13 \times 10^{16} \text{ rad/s})(0.529 \times 10^{-10} \text{ m}) = 2.19 \times 10^6 \text{ m/s}$$

This is only 0.73% the speed of light.