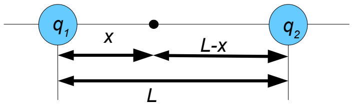
Chapter 7 Problem 102 †



Given
$L = 0.500\ m$
$q_1 = 25.0\ \mu C$
$q_2 = 45.0\ \mu C$
$k = 8.99 \times 10^9\ Nm^2/C^2$

Solution
a) At what point along the line between the two charges is the electric field zero?

Since both charges are positive, the location where their electric fields cancel is between the two charges. The electric field at a point located a distance, x, from the first charge is

$$\vec{E}_1 = \frac{kq_1}{x^2}\hat{i}$$

The electric field at the same point due to the second charge is

$$\vec{E}_2 = \frac{kq_2}{(L-x)^2}(-\hat{i})$$

Notice the direction of the electric field is in the negative direction because the second charge is to the right of the point of interest. Add the fields together and set it equal to zero.

$$\vec{E}_{tot} = \vec{E}_1 + \vec{E}_2 = 0$$

$$\frac{kq_1}{x^2}\hat{i} + \frac{kq_2}{(L-x)^2}(-\hat{i}) = 0$$

Simplify by dividing by $k\hat{i}$.

$$\frac{q_1}{x^2} - \frac{q_2}{(L-x)^2} = 0$$

$$\frac{q_1}{x^2} = \frac{q_2}{(L-x)^2}$$

Cross multiply

$$q_1(L-x)^2 = q_2x^2$$

$$(L-x)^2 = \frac{q_2}{q_1}x^2$$

$$L^2 - 2Lx + x^2 = \frac{q_2}{q_1}x^2$$

$$L^2 - 2Lx + x^2 - \frac{q_2}{q_1}x^2 = 0$$

$$L^2 - 2Lx + \left(1 - \frac{q_2}{q_1}\right)x^2 = 0$$

†Problem from Univesity Physics by Ling, Sanny and Moebs (OpenStax)

Notice this is a quadratic equation. Instead of solving it symbolically, I will substitute in the appopriate values and then solve for x.

$$(0.500\ m)^2 - 2(0.500\ m)x + \left(1 - \frac{45.0\ \mu C}{25.0 \mu C}\right) x^2 = 0$$

$$(0.250\ m^2) - (1.00\ m)x - 0.800x^2 = 0$$

Substitute into the quadratic formula and drop the units. The answer will be in meters.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-1.00) \pm \sqrt{(-1.00)^2 - 4(-0.800)(0.250)}}{2(-0.800)}$$

$$x = \frac{1.00 \pm \sqrt{1.00 + 0.800}}{-1.600} = \frac{-1.00 \mp \sqrt{1.800}}{1.600}$$

$$x = -1.464, 0.214$$

Since the answer needs to lie between the two charges, the answer is

$$x = 0.214\ m$$