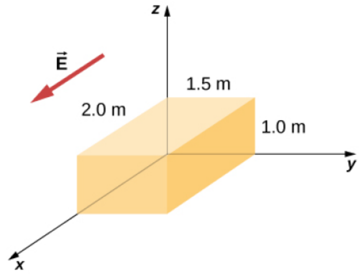
## Chapter 6 Problem 77 [†]




**Given**

$\vec{E} = \frac{a}{b+cx}\ \hat{i}$

$a = 200\ Nm/C$

$b = 2.0\ m$

$c = 2.0$

$\epsilon_0 = 8.85 \times 10^{-12}\ C^2/Nm^2$

**Solution**

Find the net charge enclosed by the shaded volume.

From Gauss's law we know that

$$\Phi = \oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\epsilon_0}$$

The shaded volume is a rectanglular box, which has 6 surfaces. We need to do the integral over each surface. Since electric field is only in the $\hat{i}$ direction, we will not see any flux going through the sides of the box that are parallel to the x-y plane and parallel to the x-z plane. The surfaces that are parallel to the y-z plane will have flux going through them.

In two-dimenions the flux integral for surfaces in the y-z plane look like

$$\Phi = \int \vec{E} d\vec{A} = \int \int (E\ \hat{i}) \cdot (dydz\hat{i})$$

The dot product between two parallel unit vectors is 1 and since the electric field function has no dependence on y or z, we can rewrite the integral as

$$\Phi = E \int dy \int dz = E\delta y \delta z = EA_{yz}$$

The y-z surface has an area of

$$A_{yz} = (1.5\ m)(1.0\ m) = 1.5\ m^2$$

When we define a surface vector in Gauss's law, it needs to be pointing outside of the volume. At $x = 0$, the outside direction is in the $-\hat{i}$ direction. The area vector is opposite of the electric field direction; therefore, we need to include a negative sign

$$\Phi_0 = -E_0 A_{yz} = -\frac{a}{b+cx} A_{yz}|_{x=0} = \frac{-aA}{b}$$

$$\Phi_0 = \frac{-(200\ Nm/C)(1.5\ m^2)}{2.0\ m} = -150\ Nm^2/C$$

[†]Problem from Univesity Physics by Ling, Sanny and Moebs (OpenStax)

Now we need to calculate the flux through the plane parallel to the y-z plane at $x = 2.0\ m$. In this case the area vector is in the positive $\hat{\imath}$ direction, so the dot product will give a positive flux.

$$\Phi_2 = E_2 A_{yz} = \frac{a}{b+cx} A_{yz}|_{x=2} = \frac{aA}{b + c(2.0\ m)}$$

$$\Phi_2 = \frac{(200\ Nm/C)(1.5\ m^2)}{(2.0\ m) + (2.0)(2.0\ m)}$$

$$\Phi_2 = 50\ Nm^2/C$$

The total flux for the box is

$$\Phi = \Phi_0 + \Phi_2 = -150\ Nm^2/C + 50\ Nm^2/C = -100\ Nm^2/C$$

Now using Gauss's law, the enclosed charge is

$$q_{enc} = \epsilon_0 \Phi = (8.85 \times 10^{-12}\ C^2/Nm^2)(-100\ Nm^2/C) = -8.85 \times 10^{-10}\ C$$

This would be $-0.885\ nC$ or $-885\ pC$.