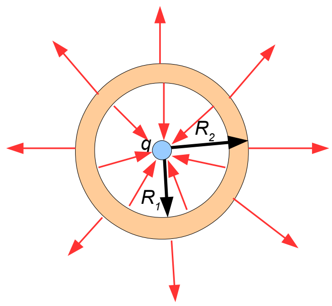
Chapter 6 Problem 70 †



Given
$q = -5.0 \times 10^{-12}\ C$
$R_1 = 3.5\ cm = 0.035\ m$
$R_2 = 4.0\ cm = 0.040\ m$
$E_2 = 8.0\ N/C$
$\epsilon_0 = 8.85 \times 10^{-12}\ C^2/Nm^2$

Solution
a) Find the charge density on the inner surface of the shell.

There are two ways you can approach this question.
Method 1: Since the shell is a conductor, there can be no electric field inside the conductor. If we place a Gaussian surface here, that implies there is no electric flux through the surface and, therefore, no net charge inside the surface. Since we know there is $-5.0 \times 10^{-12}\ C$ charge at the center, there must be $+5.0 \times 10^{-12}\ C$ charge on the inside surface of the shell. Since we want the charge per area, just divide this charge by the surface area of the spherical surface on the inside of the shell.

$$\sigma = \frac{q}{4\ \pi r^2} = \frac{5.0 \times 10^{-12}\ C}{4\ \pi (0.035\ m)^2} = 3.25 \times 10^{-10}\ C/m^2$$

Method 2: The electric field just inside the conducting shell is due to the charge at the middle of the sphere. At 3.5 cm, the electric field is

$$E = k\frac{q}{r^2} = (8.99 \times 10^9\ Nm^2/C^2)\frac{(-5.0 \times 10^{-12}\ C)}{(0.035\ m)^2} = -36.7\ N/C$$

The electric field above a conducting surface is

$$E = \frac{\sigma}{\epsilon_0}$$

then,

$$\sigma = \epsilon_0 E = (8.85 \times 10^{-12}\ C^2/Nm^2)(36.7\ N/C) = 3.25 \times 10^{-10}\ C/m^2$$

Notice I dropped the negative sign in the last calculation. The negative charge on the inside of the sphere give an electric field that is leaving the inner surface of the sphere, heading towards the center where the charge is. Since we are on the inside of the conducting sphere, the direction of the electric field is away from the surface. Therefore, the charge on the inner surface is positive.

†Problem from Univesity Physics by Ling, Sanny and Moebs (OpenStax)

b) Find the charge density on the outer surface of the sphere.

As mentioned in part a), the electric field above a conductor is

$$E = \frac{\sigma}{\epsilon_0}$$

Since the electric field just above the surface is given, the surface charge density is

$$\sigma = \epsilon_0 E = (8.85 \times 10^{-12}\ C^2/Nm^2)(8.0\ N/C) = 7.08 \times 10^{-11}\ C/m^2$$

c) Find the net charge on the conductor.

We know the surface charge density on the outer surface of the conducting shell. Multiply this by the area of the outer surface.

$$q_{outer} = \sigma 4\pi r^2 = (7.08 \times 10^{-11}\ C/m^2)4\pi(0.040\ m)^2 = 1.42 \times 10^{-12}\ C$$

Since the electric field is pointing away from the surface, this charge is positive. From part a), the charge on the inner surface is $5.0 \times 10^{-12}\ C$. Therefore, the total charge on the conductor is

$$q_{total} = q_{inner} + q_{outer} = (5.0 \times 10^{-12}\ C) + (1.42 \times 10^{-12}\ C) = 6.42 \times 10^{-12}\ C$$