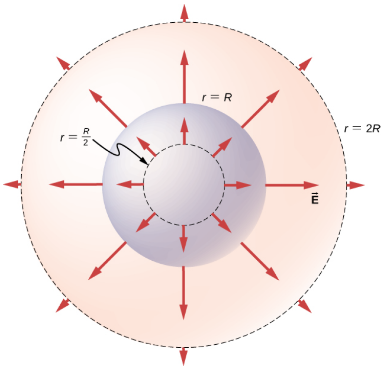
Chapter 6 Problem 44 †



Given
$Q = -30\ \mu C = -30.0 \times 10^{-6}\ C$
$R = 10.0\ cm = 0.100\ m$

Solution
a) Find the electric field at $r = 2.0\ cm$.

Since this radius is inside the spherical charge distribution, we need to determine how much of the total charge is inside this sphere. The charge density is

$$\rho = \frac{Q}{\frac{4}{3}\pi R^3} = \frac{-30.0 \times 10^{-6}\ C}{\frac{4}{3}\pi(0.100\ m)^3} = -7.16 \times 10^{-3}\ C/m^3 \qquad (Eq.1)$$

The charge inside the Gaussian surface is

$$q_{enc} = \rho V = \rho \tfrac{4}{3}\pi r^3 = (-7.16 \times 10^{-3}\ C)\tfrac{4}{3}\pi(0.020\ m)^3 = -2.40 \times 10^{-7}\ C$$

Now from Gauss' Law

$$\oint_S \vec{E} \cdot \vec{A} = \Phi = \frac{q_{enc}}{\epsilon_0}$$

We are dealing with spherical symmetry, therefore the electric field is a constant over the integral and the total surface area is that of a sphere. Therefore,

$$E 4\pi r^2 = \frac{q_{enc}}{\epsilon_0}$$

The electric field is then

$$E = \frac{q_{enc}}{4\pi\epsilon_0 r^2} \qquad (Eq.2)$$

Substituting in our values gives

$$E = \frac{-2.40 \times 10^{-7}\ C}{4\pi(8.85 \times 10^{-12}\ C^2/Nm^2)(0.020\ m)^2}$$

$$E = -5.40 \times 10^{6}\ N/C$$

The negative sign means the electric field is entering the surface not leaving. This is opposite of what is illustrated in the provided diagram.

†Problem from Univesity Physics by Ling, Sanny and Moebs (OpenStax)

b) What is the electric field at $r = 5.0\ cm$.

Find the charge inside the surface by multiplying the charge density calculated in Eq. (1) and multiply by the volume.

$$q_{enc} = \rho V = \rho \tfrac{4}{3}\pi r^3 = (-7.16 \times 10^{-3}\ C)\tfrac{4}{3}\pi(0.050\ m)^3 = 2.40 \times 10^{-7}\ C = -3.75 \times 10^{-6}\ C$$

Using Eq. (2) we have an electric field of

$$E = \frac{-3.75 \times 10^{-6}\ C}{4\pi(8.85 \times 10^{-12}\ C^2/Nm^2)(0.050\ m)^2}$$

$$E = -1.35 \times 10^{7}\ N/C$$

c) What is the electric field at $r = 20.0\ cm$?

This distance is beyond the surface of the charge distribution. Therefore, the enclosed charge is the same as the total charge. Now use Eq. (2) and calculate the electric field.

$$E = \frac{-3.0 \times 10^{-5}\ C}{4\pi(8.85 \times 10^{-12}\ C^2/Nm^2)(0.20\ m)^2}$$

$$E = -6.74 \times 10^{6}\ N/C$$