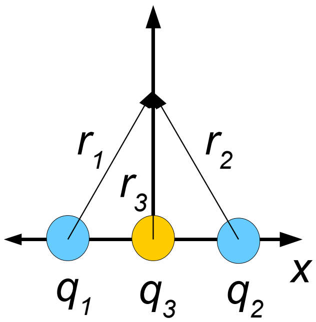
**Chapter 5 Problem 79** [†]




**Given**
$k = 8.99 \times 10^9 \; \frac{Nm^2}{C^2}$
$q_1 = q_2 = 4.00 \times 10^{-6} \; C$
$\vec{p}_1 = -3.0 \; m \; \hat{i}$
$\vec{p}_2 = 3.0 \; m \; \hat{i}$
$\vec{p}_3 = 0$
$\vec{p} = 3.0 \; m \; \hat{j}$

**Solution**
Find the value of a charge placed at the origin that results in no electric field at $\vec{r} = 3.0 \; m \; \hat{j}$.

The electric field due to a combination of point charges is given by

$$\vec{E} = \Sigma k \frac{q_i}{r_i^2} \hat{r}_i$$

There are three charges and their net electric field must add up to zero.

$$\vec{E} = k\frac{q_1}{r_1^2}\hat{r}_1 + k\frac{q_2}{r_2^2}\hat{r}_2 + k\frac{q_3}{r_3^2}\hat{r}_3 = 0 \qquad (Eq.1)$$

To find $\vec{r}_1$ subtract the position of the point of interest from the position of $q_1$.

$$\vec{r}_1 = \vec{p} - \vec{p}_1 = 3.0\hat{j} \; m - (-3.0\hat{i} \; m) = \{3.0\hat{i} + 3.0\hat{j}\} \; m$$

The magnitude is

$$r_1 = \sqrt{(3.0)^2 + (3.0)^2} \; = 4.24 \; m$$

The unit vector is

$$\hat{r}_1 = \frac{\vec{r}_1}{r_1} = \frac{3.0\hat{i} + 3.0\hat{j}}{4.24 \; m} = 0.707\hat{i} + 0.707\hat{j}$$

Notice that this unit vector corresponds to $\cos 45°\hat{i} + \sin 45°\hat{j}$.
Repeat this for $q_2$.

$$\vec{r}_2 = 3.0\hat{j} \; m - (3.0\hat{i} \; m) = \{-3.0\hat{i} + 3.0\hat{j}\} \; m$$

[†]Problem from Univesity Physics by Ling, Sanny and Moebs (OpenStax)

$$r_2 = \sqrt{(-3.0)^2 + (3.0)^2} = 4.24\ m$$

The unit vector is

$$\hat{r}_2 = \frac{-3.0\hat{\imath} + 3.0\hat{\jmath}}{4.24\ m} = -0.707\hat{\imath} + 0.707\hat{\jmath}$$

Since $q_3$ is at the origin, then

$$\vec{r}_3 = 3.0\hat{\jmath}\ m$$

$$r_3 = 3.0\ m$$

$$\hat{r}_3 = \hat{\jmath}$$

Substitute into equation (1) and solve for $q_3$.

$$k\frac{4.00 \times 10^{-6}\ C}{(4.24\ m)^2}(0.707\hat{\imath} + 0.707\hat{\jmath}) + k\frac{4.00 \times 10^{-6}\ C}{(4.24\ m)^2}(-0.707\hat{\imath} + 0.707\hat{\jmath}) + k\frac{q_3}{(3.0\ m)^2}\hat{\jmath} = 0$$

In the x-direction

$$k\frac{4.00 \times 10^{-6}\ C}{(4.24\ m)^2}(0.707) + k\frac{4.00 \times 10^{-6}\ C}{(4.24\ m)^2}(-0.707) = 0$$

Since $q_1$ and $q_2$ are the same value and are equi-distant from the origin, the x-component of the electric fields cancel out.

In the y-direction

$$k\frac{4.00 \times 10^{-6}\ C}{(4.24\ m)^2}(0.707) + k\frac{4.00 \times 10^{-6}\ C}{(4.24\ m)^2}(0.707) + k\frac{q_3}{(3.0\ m)^2} = 0$$

Since all of the term are multiplied by $k$, I will divide both sides by this value to get rid of it. Now, solving for $q_3$ gives

$$\frac{4.00 \times 10^{-6}\ C}{(4.24\ m)^2}(0.707) + \frac{4.00 \times 10^{-6}\ C}{(4.24\ m)^2}(0.707) = -\frac{q_3}{(3.0\ m)^2}$$

Simplifying each term

$$1.57 \times 10^{-7}\ C/m^2 + 1.57 \times 10^{-7}\ C/m^2 = -\frac{q_3}{(3.0\ m)^2}$$

$$q_3 = -(3.0\ m)^2(3.14 \times 10^{-7}\ C/m^2) = -2.83 \times 10^{-6}\ C$$