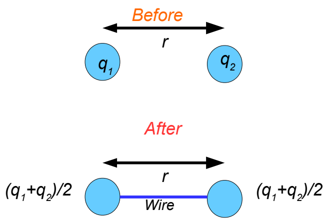
Chapter 5 Problem 58 †



Given

$k = 8.99 \times 10^9 \ \frac{Nm^2}{C^2}$
$r = 0.25 \ m$
$F_B = 0.050 \ N$
$F_A = 0.060 \ N$

Solution

Find the original charge on each sphere.

The electric force between the two spheres is given by Coulomb's law. F_B is used to indicate the force before the spheres are connected by a wire.

$$F_B = k\frac{q_1 q_2}{r^2} \qquad (Eq.1)$$

After the balls are connected by the wire, the remaining charge is distributed evenly between the two spheres. This will result in a final force of F_A.

$$F_A = k\frac{((q_1+q_2)/2)((q_1+q_2)/2)}{r^2}$$

Simplifying the second equation gives

$$F_A = k\frac{(q_1+q_2)^2}{4r^2}$$

Take the equation for F_B and solve for q_2.

$$q_2 = \frac{F_B r^2}{k q_1} \qquad (Eq.2)$$

Substitute this into the equation for F_A.

$$F_A = k\frac{(q_1 + (\frac{F_B r^2}{k q_1}))^2}{4r^2}$$

$$\frac{4F_A r^2}{k} = (q_1 + (\frac{F_B r^2}{k q_1}))^2$$

$$\pm\sqrt{\frac{4F_A r^2}{k}} = q_1 + (\frac{F_B r^2}{k q_1})$$

To make the math easier, let

$$A = \sqrt{\frac{4F_A r^2}{k}}$$

†Problem from Univesity Physics by Ling, Sanny and Moebs (OpenStax)

$$B = \frac{F_B r^2}{k}$$

and substitute in the know values

$$A = \sqrt{\frac{4(0.060\ N)(0.25\ m)^2}{8.99 \times 10^9\ \frac{Nm^2}{C^2}}} = 1.29 \times 10^{-6}\ C$$

$$B = \frac{(0.050\ N)(0.25\ m)^2}{8.99 \times 10^9\ \frac{Nm^2}{C^2}} = 3.47 \times 10^{-13}\ C^2$$

Then,

$$\pm A = q_1 + \frac{B}{q_1}$$

Multiply both sides by q_1 and put into a form that can be solved using the quadratic formula.

$$0 = q_1^2 \pm Aq_1 + B$$

Solutions for the equation with a positive second term gives

$$0 = q_1^2 + (1.29 \times 10^{-6})q_1 + (3.47 \times 10^{-13})$$

$$q_1 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(1.29 \times 10^{-6}) \pm \sqrt{(1.29 \times 10^{-6})^2 - 4(1)(3.47 \times 10^{-13})}}{2(1)}$$

$$q_1 = \frac{-(1.29 \times 10^{-6}) \pm (5.25 \times 10^{-7})}{2}$$

$$q_1 = -3.83 \times 10^{-7}, -9.08 \times 10^{-7}$$

Solutions for the equation with a negative second term gives

$$0 = q_1^2 - (1.29 \times 10^{-6})q_1 + (3.47 \times 10^{-13})$$

$$q_1 = \frac{-(-1.29 \times 10^{-6}) \pm \sqrt{(-1.29 \times 10^{-6})^2 - 4(1)(3.47 \times 10^{-13})}}{2(1)}$$

$$q_1 = 9.08 \times 10^{-7}, 3.83 \times 10^{-7}\ C$$

Notice each set of solutions have the same magnitudes, but with opposite sides. By equation (2), we also see that q_2 will have the same sign as q_1. That should make sense since the spheres repel each other. The charges can either be both positive or both negative.
Let's use the first positive solution and substitute into equation (2)

$$q_2 = \frac{F_B r^2}{kq_1} = \frac{(0.050\ N))(0.25\ m)^2}{(8.99 \times 10^9\ \frac{Nm^2}{C^2})(9.08 \times 10^{-7}\ C)} = 3.82 \times 10^{-7}\ C$$

Notice the answer is the other solution to q_1. That should make sense because interchanging q_1 and q_2 will not change the problem.