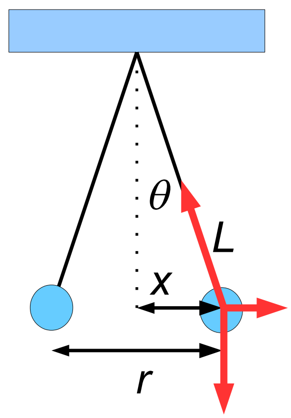
Chapter 5 Problem 55 †



Given
$k = 8.99 \times 10^9 \ \frac{Nm^2}{C^2}$
$g = 9.80 \ m/s^2$
$m = 5.0 \ g = 5.0 \times 10^{-3} \ kg$
$L = 50 \ cm = 0.500 \ m$
$\theta = 5.0°$

Solution
Find the magnitude of the charge on each ball.

First a free-body diagram needs to be generated. There are three forces: one due to electric repulsion to the right, one due to gravity in the downward direction and the last due to tension in the string to the upper-left. Using the angle designated in the diagram and let the positive x-axis be to the right and positive y-axis in the upward direction, then by Newton's 2nd law we have the following equation.

$$\Sigma \vec{F}_i = ma = \vec{F}_T + \vec{F}_e + \vec{F}_g$$

Now

$$\vec{F}_T = -T\sin\theta\hat{\imath} + T\cos\theta\hat{\jmath}$$

$$\vec{F}_e = k\frac{q_1 q_2}{r^2}\hat{\imath} = k\frac{QQ}{r^2}\hat{\imath} = k\frac{Q^2}{r^2}\hat{\imath}$$

$$\vec{F}_g = -mg\hat{\jmath}$$

Substitute the individual forces into Newton's 2nd law and set the acceleration equal to zero (this is a statics problem).

$$ma = 0 = -T\sin\theta\hat{\imath} + T\cos\theta\hat{\jmath} + k\frac{Q^2}{r^2}\hat{\imath} - mg\hat{\jmath}$$

In the x-direction we get

$$0 = -T\sin\theta + k\frac{Q^2}{r^2}$$

†Problem from Univesity Physics by Ling, Sanny and Moebs (OpenStax)

or

$$T \sin\theta = k\frac{Q^2}{r^2}$$

In the y-direction we get

$$0 = T\cos\theta - mg$$

or

$$T\cos\theta = mg$$

Dividing the x-direction equation by the y-direction equation we get

$$\frac{T\sin\theta}{T\cos\theta} = \frac{k\frac{Q^2}{r^2}}{mg}$$

Simplifying this equation gives

$$\frac{\sin\theta}{\cos\theta} = \tan\theta = k\frac{Q^2}{mgr^2}$$

Solving for Q gives

$$Q^2 = \frac{mgr^2\tan\theta}{k}$$

$$Q = \sqrt{\frac{mgr^2\tan\theta}{k}}$$

Although we know the angle θ, we need to determine the distance between the two balls. By trigonometry we have

$$\sin\theta = \frac{x}{L}$$

The distance x is

$$x = L\sin\theta = (0.500\ m)\sin(5.00^\circ) = 0.0436\ m$$

The distance between the two balls is twice this

$$r = 2x = 2(0.0436\ m) = 0.0872\ m$$

Now substitute into the equation for finding the value of Q.

$$Q = \sqrt{\frac{(5.0\times10^{-3}\ kg)(9.80\ m/s^2)(0.0872\ m)^2\tan(5.00^\circ)}{8.99\times10^9\ \frac{Nm^2}{C^2}}}$$

$$Q = 6.02\times10^{-8}\ C$$

This is 60.2 nC. Since the balls repel each other, they have charge of the same sign. Both are either negative or both are positive.