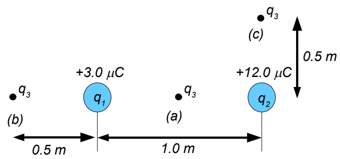
Chapter 5 Problem 49 †



Given

$k = 8.99 \times 10^9 \ \frac{Nm^2}{C^2}$
$q_1 = 3.00\ \mu C = 3.0 \times 10^{-6}\ C$
$q_2 = 12.00\ \mu C = 12.0 \times 10^{-6}\ C$
$q_3 = -2.00\ nC = -2.0 \times 10^{-9}\ C$

Solution

a) Find the magnitude and direction of the force on the third charge when halfway between the first two charges.

The total force on the third charge is

$$\vec{F}_3 = k\frac{q_1q_3}{r_{31}^2}\hat{r}_{31} + k\frac{q_2q_3}{r_{32}^2}\hat{r}_{32}$$

The unit vector between q_1 and q_3 is in the positive $\hat{i}$ direction. The unit vector between q_2 and q_3 is in the negative $\hat{i}$ direction. Now substitute in the charge values and the distances.

$$\vec{F}_3 = (8.99{\times}10^9\ \tfrac{Nm^2}{C^2})\frac{(3.0 \times 10^{-6}\ C)(-2.0 \times 10^{-9}\ C)}{(0.50\ m)^2}(\hat{i}){+}(8.99{\times}10^9\ \tfrac{Nm^2}{C^2})\frac{(12.0 \times 10^{-6}\ C)(-2.0 \times 10^{-9}\ C)}{(0.50\ m)^2}(-\hat{i})$$

$$\vec{F}_3 = 2.16 \times 10^{-4}\ N(-\hat{i}) + 8.63 \times 10^{-4}\ N(\hat{i})$$

$$\vec{F}_3 = 6.47 \times 10^{-4}\ N(\hat{i})$$

The force is $6.47 \times 10^{-4}\ N$ in the positive x-direction.

b) Find the magnitude and direction of the force on the third charge when it is $0.50\ m$ to the left of the first charge.

The total force on the third charge is

$$\vec{F}_3 = k\frac{q_1q_3}{r_{31}^2}\hat{r}_{31} + k\frac{q_2q_3}{r_{32}^2}\hat{r}_{32}$$

The unit vector between q_1 and q_3 is in the negative $\hat{i}$ direction. The unit vector between q_2 and q_3 is in the negative $\hat{i}$ direction. Now substitute in the charge values and the distances.

$$\vec{F}_3 = (8.99{\times}10^9\ \tfrac{Nm^2}{C^2})\frac{(3.0 \times 10^{-6}\ C)(-2.0 \times 10^{-9}\ C)}{(0.50\ m)^2}(-\hat{i}){+}(8.99{\times}10^9\ \tfrac{Nm^2}{C^2})\frac{(12.0 \times 10^{-6}\ C)(-2.0 \times 10^{-9}\ C)}{(1.50\ m)^2}(-\hat{i})$$

$$\vec{F}_3 = 2.16 \times 10^{-4}\ N(\hat{i}) + 0.959 \times 10^{-4}\ N(\hat{i})$$

$$\vec{F}_3 = 3.12 \times 10^{-4}\ N(\hat{i})$$

The force is $3.12 \times 10^{-4}\ N$ in the positive x-direction.

†Problem from Univesity Physics by Ling, Sanny and Moebs (OpenStax)

c) Find the magnitude and direction of the force on the third charge when it is 0.50 m above the second charge.

The total force on the third charge is

$$\vec{F}_3 = k\frac{q_1 q_3}{r_{31}^2}\hat{r}_{31} + k\frac{q_2 q_3}{r_{32}^2}\hat{r}_{32}$$

The unit vector between q_2 and q_3 is in the positive $\hat{j}$ direction. The unit vector between q_1 and q_3 is a little more complicated. First determine the vector pointing from q_1 to q_3 and then make it a unit vector.

$$\vec{r}_{31} = 1.00\hat{i} + 0.500\hat{j}\ m$$

The unit vector is then

$$\hat{r}_{31} = \frac{\vec{r}_{31}}{||\vec{r}_{31}||} = \frac{1.00\hat{i} + 0.500\hat{j}\ m}{\sqrt{(1.00\ m)^2 + (0.500\ m)^2}} = 0.894\hat{i} + 0.447\hat{j}$$

Now substitute in the charge values and the distances. Notice that the distance between q_1 and q_3 is $\sqrt{(1.0)^2 + (0.5)^2} = 1.12\ m$.

$$\begin{aligned}\vec{F}_3 &= (8.99 \times 10^9\ \tfrac{Nm^2}{C^2})\frac{(3.0 \times 10^{-6}\ C)(-2.0 \times 10^{-9}\ C)}{(1.12\ m)^2}(0.894\hat{i} + 0.447\hat{j}) \\ &+ (8.99 \times 10^9\ \tfrac{Nm^2}{C^2})\frac{(12.0 \times 10^{-6}\ C)(-2.0 \times 10^{-9}\ C)}{(0.500\ m)^2}(\hat{j})\end{aligned} \tag{1}$$

$$\vec{F}_3 = 4.30 \times 10^{-5}\ N(-0.894\hat{i} - 0.447\hat{j}) + 8.63 \times 10^{-4}\ N(-\hat{j})$$

$$\vec{F}_3 = -3.84 \times 10^{-5}\ N(\hat{i}) - 1.92 \times 10^{-5}\ N(\hat{j}) - 8.63 \times 10^{-4}\ N(\hat{j})$$

$$\vec{F}_3 = -3.84 \times 10^{-5}\ N(\hat{i}) - 8.82 \times 10^{-4}\ N(\hat{j})$$

The magnitude of the total force is

$$F_3 = \sqrt{(-3.84 \times 10^{-5})^2 + (-8.82 \times 10^{-4})^2}\ N = 8.83 \times 10^{-4}\ N$$

The direction (angle) of the total force is

$$\phi = \tan^{-1}\left(\frac{-8.82 \times 10^{-4}\ N}{-3.84 \times 10^{-5}\ N}\right) = \tan^{-1}(23.0) = 87.5°$$

Notice that the vector is pointed towards the third quadrant, so the angle given should be increased by 180°. Therefore the angle is 267.5° counter-clockwise from the positive x-axis. This is the same at 66.5° below the negative x-axis.