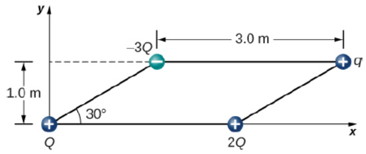
Chapter 5   Problem 118 [†]



Given

$k = 8.99 \times 10^9 \ \frac{Nm^2}{C^2}$
$Q = 8.00\ \mu C = 8.00 \times 10^{-6}\ C$
$q = 5.00\ \mu C = 5.00 \times 10^{-6}\ C$

Solution

a) Find the electric field at the unoccupied corner of the parallelogram.

The electric field for a collection of point charges is given by

$$\vec{E} = \Sigma \frac{kq_i}{r_i^2}\hat{r}_i$$

In this problem there are three charges.

$$\vec{E} = k\frac{q_1}{r_1^2}\hat{r}_1 + k\frac{q_2}{r_2^2}\hat{r}_2 + k\frac{q_3}{r_3^2}\hat{r}_3$$

Now comes the hard part of determining the displacement vectors between each of the charges and the point of interest. First let's get the coordinates of each of the points, where $\vec{p}_1$ is the location of the first charge and likewise for the 2nd and 3rd charge. If the subscript is left off, it is refering to the point of interest.

Charge 1 is at the origin so

$$\vec{p}_1 = 0$$

Charge 2 is 30° to the upper right from the origin. Since the length of the hypotenuse is not given, we need to use the opposite side and the angle to determine the adjacent side, which is the x-coordinate. We can use the tangent function for this

$$\tan\theta = \frac{opp}{adj}$$

$$adj = \frac{opp}{\tan\theta} = \frac{1.00\ m}{\tan 30°} = 1.73\ m$$

Therefore the position of charge 2 is

$$\vec{p}_2 = \{1.73\hat{\imath} + 1.00\hat{\jmath}\}\ m$$

The coordinate for the third charge is

$$\vec{p}_3 = 3.00\hat{\imath}\ m$$

and the coordinate of the point of interest is

$$\vec{p} = 1.73\hat{\imath} + 1.00\hat{\jmath} + 3.00\hat{\imath} = \{4.73\hat{\imath} + 1.00\hat{\jmath}\}\ m$$

[†]Problem from Univesity Physics by Ling, Sanny and Moebs (OpenStax)

Now the displacements between each of the charges as the point of interest are

$$q_1 = Q$$

$$\vec{r}_1 = \vec{p} - \vec{p}_1 = 4.73\hat{\imath} + 1.00\hat{\jmath} - 0 = \{4.73\hat{\imath} + 1.00\hat{\jmath}\}\ m$$

$$r_1 = \sqrt{(4.73)^2 + (1.00)^2} = 4.83\ m$$

$$\hat{r}_1 = \frac{\vec{r}_1}{r_1} = \frac{\{4.73\hat{\imath} + 1.00\hat{\jmath}\}\ m}{4.83\ m} = 0.979\hat{\imath} + 0.207\hat{\jmath}$$

$$q_2 = -3Q$$

$$\vec{r}_2 = \vec{p} - \vec{p}_2 = 4.73\hat{\imath} + 1.00\hat{\jmath} - \{1.73\hat{\imath} + 1.00\hat{\jmath}\} = 3.00\hat{\imath}\ m$$

$$r_2 = \sqrt{(3.00)^2 + (0)^2} = 3.00\ m$$

$$\hat{r}_2 = \frac{\vec{r}_2}{r_2} = \frac{3.00\hat{\imath}\ m}{3.00\ m} = \hat{\imath}$$

$$q_3 = 2Q$$

$$\vec{r}_3 = \vec{p} - \vec{p}_3 = 4.73\hat{\imath} + 1.00\hat{\jmath} - 3.00\hat{\imath} = \{1.73\hat{\imath} + 1.00\hat{\jmath}\}\ m$$

$$r_3 = \sqrt{(1.73)^2 + (1.00)^2} = 2.00\ m$$

$$\hat{r}_3 = \frac{\vec{r}_3}{r_3} = \frac{\{1.73\hat{\imath} + 1.00\hat{\jmath}\}\ m}{2.00\ m} = 0.865\hat{\imath} + 0.500\hat{\jmath}$$

Now substitute into the equation for the electric field

$$\vec{E} = k\frac{q_1}{r_1^2}\hat{r}_1 + k\frac{q_2}{r_2^2}\hat{r}_2 + k\frac{q_3}{r_3^2}\hat{r}_3$$

$$\vec{E} = k\frac{Q}{(4.83\ m)^2}(0.979\hat{\imath} + 0.207\hat{\jmath}) + k\frac{-3Q}{(3.00\ m)^2}\hat{\imath} + k\frac{2Q}{(2.00\ m)^2}(0.865\hat{\imath} + 0.500\hat{\jmath})$$

Factor k and Q out and simplify

$$\vec{E} = kQ\left(\frac{1}{(4.83\ m)^2}(0.979\hat{\imath} + 0.207\hat{\jmath}) + \frac{-3}{(3.00\ m)^2}\hat{\imath} + \frac{2}{(2.00\ m)^2}(0.865\hat{\imath} + 0.500\hat{\jmath})\right)$$

$$\vec{E} = kQ\left(\frac{0.979\hat{\imath} + 0.207\hat{\jmath}}{(4.83\ m)^2} + \frac{-3\hat{\imath}}{(3.00\ m)^2} + \frac{1.73\hat{\imath} + 1.00\hat{\jmath}}{(2.00\ m)^2}\right)$$

$$\vec{E} = kQ\left(\frac{0.04197\hat{\imath} + 0.00887\hat{\jmath}}{(1\ m)^2} + \frac{-0.33333\hat{\imath}}{(1\ m)^2} + \frac{0.4325\hat{\imath} + 0.25000\hat{\jmath}}{(1\ m)^2}\right)$$

$$\vec{E} = kQ\left(\frac{0.14114\hat{\imath} + 0.25887\hat{\jmath}}{(1\ m)^2}\right)$$

Now substitute in the values for k and Q

$$\vec{E} = (8.99 \times 10^9\ \tfrac{Nm^2}{C^2})(8.00 \times 10^{-6}\ C)\left(\frac{0.14114\hat{\imath} + 0.25887\hat{\jmath}}{(1\ m)^2}\right)$$

$$\vec{E} = \{10,150\hat{\imath} + 18,620\hat{\jmath}\}\ N/C$$

This field has a magnitude of $21,200\ N/C$ at an angle of $61.4°$ counter-clockwise from the positive x-axis.

b) What is the force on a $5.0\ \mu C$ charge placed at the unoccupied corner of the parallelogram?

The relationship between electric field and force is

$$\vec{F} = q\vec{E}$$

The force is then

$$\vec{F} = q\vec{E} = (5.00 \times 10^{-6}\ C)(\{10,150\hat{\imath} + 18,620\hat{\jmath}\}\ N/C) = \{0.0508\hat{\imath} + 0.0931\hat{\jmath}\}\ N$$

The magnitude of the force is $0.106\ N$ at an angle of $61.4°$ counter-clockwise from the positive x-axis.