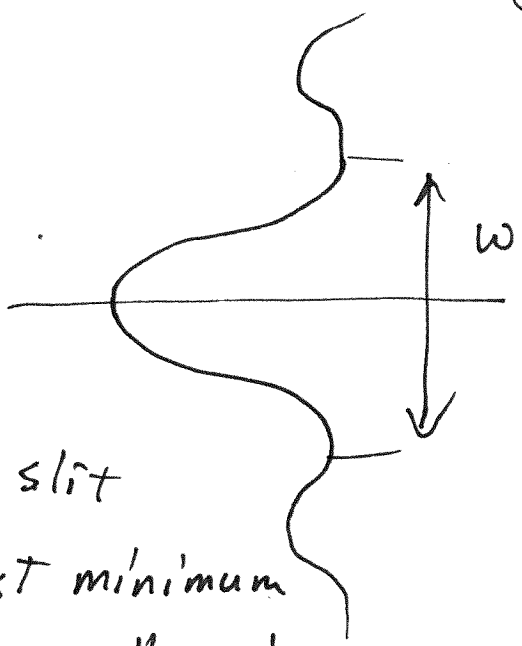
Ch.4 Prob. 32

$w = 5.0\,mm$

$\lambda = 600\,nm$

$L = 2.0\,m$



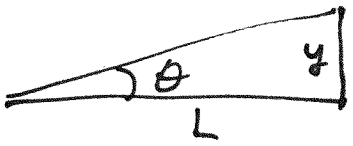
a) What is the width of the slit

The distance of the first minimum from the center of the pattern is

$$y = \frac{w}{2} = \frac{5.0\,mm}{2} = \underline{2.5 \times 10^{-3}\,m}$$

Now find the angle

$$\tan\theta = \frac{y}{L}$$



$$\theta = \tan^{-1}\left(\frac{y}{L}\right) = \tan^{-1}\left(\frac{2.5 \times 10^{-3}\,m}{2.0\,m}\right) = 0.0716^\circ$$

Now use the single-slit formula where $m = 1$

$$D \sin\theta = m\lambda = \lambda$$

$$D = \frac{\lambda}{\sin\theta} = \frac{600 \times 10^{-9}\,m}{\sin(0.0716)}$$

$$D = 4.8 \times 10^{-4}\,m = \boxed{0.48\,mm}$$

b) Find the ratio of the intensity at 4.5 mm compared to the intensity at the center.

Begin with the intensity formula for the single-slit.

$$I = I_0 \left(\frac{\sin\beta}{\beta}\right)^2 \quad \text{where } \beta = \frac{\phi}{2} = \frac{\pi D \sin\theta}{\lambda}$$

At the center $\theta = 0$. This gives us a problem where $\frac{\sin 0}{0}$. However, at the center, the intensity is I_0

$$\therefore \frac{I(4.5\text{mm})}{I(0)} = \frac{I_0\left(\frac{\sin\beta}{\beta}\right)^2}{I_0} = \left(\frac{\sin\beta}{\beta}\right)^2$$

so we need to solve for β

$$\beta = \frac{\pi D \sin\theta}{\lambda}$$

by the small angle approx. we can replace $\sin\theta$ with $\tan\theta = \frac{y}{L}$

$$\beta = \frac{\pi D y}{\lambda L} = \frac{\pi (4.8\times10^{-4}\text{m})(4.5\times10^{-3}\text{m})}{(600\times10^{-9}\text{m})(2.0\text{m})}$$

$$\beta = 5.65$$

so $$\frac{I(4.5\text{mm})}{I(0)} = \left[\frac{\sin(5.65)}{5.65}\right]^2 = [0.0174]^2 = \boxed{3.04\times10^{-4}}$$