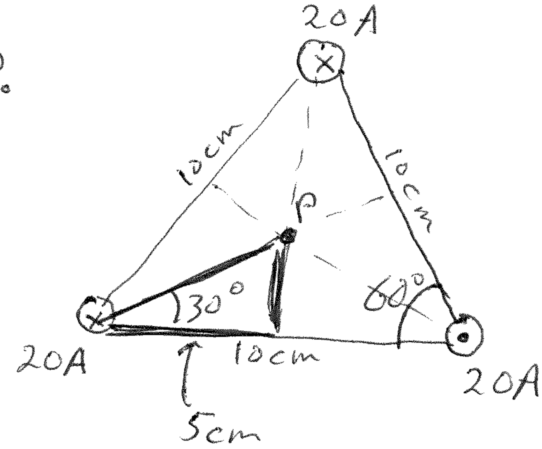
Ch. 12 Prob. 65

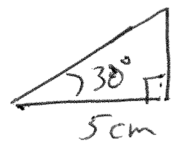
#1



Find the magnitude of the B-field at P.

Each wire is equidistant from P.
That distance can be calculated using Trigonometry.

The small right triangle in the lower left corner is



The hypotenuse is

$$\cos\theta = \frac{adj}{hyp} \longrightarrow hyp = \frac{adj}{\cos\theta} = \frac{5\text{ cm}}{\cos 30°} = 5.77\text{ cm}$$

for each wire

$$B_1 = \frac{\mu_0 I}{2\pi r} = \frac{\left(4\pi\times10^{-7}\frac{Tm}{A}\right)(20A)}{2\pi\,(5.77\times10^{-2}\,m)} = \underline{6.93\times10^{-5}\,T}$$

Each wire's magnetic field must be add as a vector
since the direction is given by Biot-Savart's Law

$$d\vec{B} = \frac{\mu_0 I}{2\pi}\frac{d\vec{s}\times\hat{r}}{r^2}$$

The current is either into or out of the page $\therefore$ it is either in the ~~+back~~ $+\hat{k}$ or $-\hat{k}$ direction.

Since we ~~are to~~ have already determined the magnitude, all we need is the direction.

Bottom left wire

$d\vec{s} = -dz\,\hat{k}$ and $\hat{r} = \cos 30\,\hat{i} + \sin 30\,\hat{j}$

Then $(-\hat{k})\times(\cos 30\,\hat{i} + \sin 30\,\hat{j}) = \cos 30(-\hat{k}\times\hat{i}) + \sin 30(-\hat{k}\times\hat{j})$

$= \cos 30(-\hat{j}) + \sin 30(\hat{i})$

The B-field vector is then

$$\vec{B}_{BL} = 6.93\times10^{-5}\,T(\sin 30\,\hat{i} - \cos 30\,\hat{j})$$

$$\underline{\vec{B}_{BL} = (3.465\,\hat{i} - 6.000\,\hat{j})\times10^{-5}\,T}$$

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#2

Bottom Right wire

$d\vec{s} = dz\hat{k}$ and $\hat{r} = -\cos\theta\,\hat{i} + \sin 30\,\hat{j}$.

Then $(\hat{k}) \times (-\cos\theta\,\hat{i} + \sin 30\,\hat{j}) = \cos\theta(\hat{k}\times(-\hat{i})) + \sin\theta(\hat{k}\times\hat{j})$

$= \cos\theta(-\hat{j}) + \sin\theta(-\hat{i})$

The B-field vector is Then

$\vec{B}_{BR} = (6.93\times10^{-5}\,T)(-\sin 30\,\hat{i} - \cos 30\,\hat{j}) = \underline{(-3.465\,\hat{i} - 6.000\,\hat{j})\times10^{-5}\,T}$

Top wire

$d\vec{s} = -dz\hat{k}$ and $\hat{r} = -\hat{j}$

Then $(-\hat{k})\times(-\hat{j}) = -\hat{i}$

The B-field vector is Then

$\vec{B}_T = -6.93\times10^{-5}\,\hat{i}\,T$

combining These give.

$\vec{B}_{total} = (3.465\,\hat{i} - 6.000\,\hat{j})\times10^{-5}\,T + (-3.465\,\hat{i} - 6.000\,\hat{j})\times10^{-5}\,T$
$- 6.93\times10^{-5}\,\hat{i}\,T$

$= [(3.465 - 3.465 - 6.93)\hat{i} + (-6.000 - 6.000)\hat{j}]\times10^{-5}\,T$

$= (-6.93\,\hat{i} - 12.00\,\hat{j})\times10^{-5}\,T$

$B_{total} = \sqrt{(-6.93)^2 + (-12.00)^2}\times10^{-5}\,T = \cancel{13.8}\ 13.9\times10^{-5}\,T$

$\theta = \tan^{-1}\left(\frac{-12.00}{-6.93}\right) = 60°$ (But This should be in The 3rd Quadrant)

$\therefore\ \boxed{\vec{B}_{total} = 13.9\times10^{-5}\,T\ \angle 240°\ \text{or}\ 60°\ \text{south of west}}$