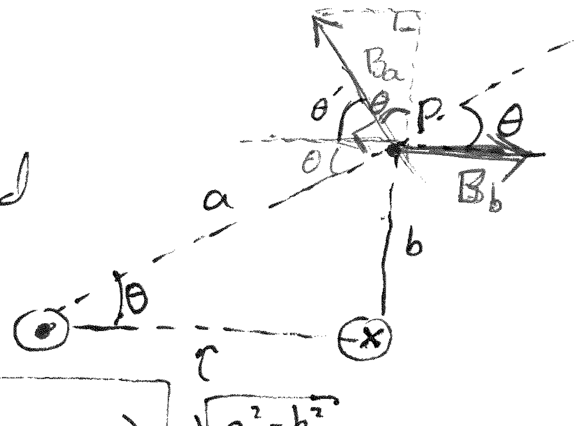
Ch.12 Prob. 33

Find the magnetic field at P is

By the right-hand rule the B field for each wire is illustrated to the right in red.



~~B_a~~

$$\vec{B}_a = \frac{\mu_0 I}{2\pi a}\left(\sin\theta\,(-\hat{i}) + \cos\theta\,(\hat{j})\right)$$

$$\vec{B}_b = \frac{\mu_0 I}{2\pi b}(+\hat{i})$$

Now $\cos\theta = \frac{\sqrt{a^2-b^2}}{a}$

$\sin\theta = \frac{b}{a}$

$$\vec{B}_{tot} = \vec{B}_a + \vec{B}_b = \frac{\mu_0 I}{2\pi a}\left[-\frac{b}{a}\hat{i} + \frac{\sqrt{a^2-b^2}}{a}\hat{j}\right] + \frac{\mu_0 I}{2\pi b}\hat{i}$$

$$\boxed{\vec{B}_{tot} = \frac{\mu_0 I}{2\pi}\left[\frac{-b}{a^2} + \frac{1}{b}\right]\hat{i} + \frac{\mu_0 I\sqrt{a^2-b^2}}{2\pi a^2}\hat{j}}$$

The magnitude of the B_{tot} is

$$B = \sqrt{\left(\frac{\mu_0 I}{2\pi}\right)^2\left[\frac{-b}{a^2} + \frac{1}{b}\right]^2 + \left(\frac{\mu_0 I}{2\pi}\right)^2\left[\frac{\sqrt{a^2-b^2}}{a^2}\right]^2}$$

$$= \frac{\mu_0 I}{2\pi}\sqrt{\left(\frac{-b}{a^2}\right)^2 + \frac{-2b}{a^2 b} + \frac{1}{b^2} + \frac{a^2-b^2}{a^4}}$$

$$= \frac{\mu_0 I}{2\pi}\sqrt{\frac{+b^2}{a^4} - 2\frac{a^2}{a^4} + \frac{1}{b^2} + \frac{a^2}{a^4} - \frac{b^2}{a^4}}$$

$$= \frac{\mu_0 I}{2\pi}\sqrt{\frac{1}{b^2} - \frac{a^2}{a^4}} = \frac{\mu_0 I}{2\pi}\sqrt{\frac{1}{b^2} - \frac{1}{a^2}}$$

$$= \frac{\mu_0 I}{2\pi}\sqrt{\frac{a^2-b^2}{a^2 b^2}} = \frac{\mu_0 I}{2\pi ab}\sqrt{a^2-b^2}$$

$$\boxed{B = \frac{\mu_0 I}{2\pi ab}\sqrt{a^2-b^2}}$$