Ch.12 Pr 17

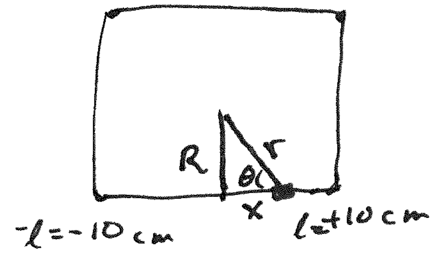
0.01 cm connectors at corners has minimal effect.

$I = 10\,A$

$L = 20\,cm$

$\ell = 10\,cm$

$R = 10\,cm$



Solve the problem for a straight wire from $-\ell$ to $+\ell$ and then multiple by 4 to find the total field strength.

$$\boxed{r = \sqrt{x^2+R^2} \qquad \sin\theta = \frac{R}{\sqrt{x^2+R^2}}}$$

From Biot-Savarts law

$$\vec{B}_1 = \int_{-\ell}^{+\ell} \frac{\mu_0 I}{4\pi} \frac{d\vec{\ell} \times \hat{r}}{r^2} = \frac{\mu_0 I}{4\pi} \int_{-\ell}^{+\ell} \frac{dx\, \hat{i} \times \hat{r}}{r^2} = \frac{\mu_0 I}{4\pi} \int_{-\ell}^{+\ell} \frac{dx \sin\theta}{(x^2+R^2)}$$

$$= \frac{\mu_0 I}{4\pi} \int_{-\ell}^{+\ell} \frac{R\hat{k}\,dx}{(x^2+R^2)^{3/2}} = \frac{\mu_0 I R \hat{k}}{4\pi} \int_{-\ell}^{+\ell} \frac{dx}{(x^2+R^2)^{3/2}}$$

Solve the integral (see p.541 eq.12.7)

$$\vec{B}_1 = \frac{\mu_0 I R \hat{k}}{4\pi} \left(\frac{x}{R^2(x^2+R^2)^{1/2}} \right) \Bigg|_{-\ell}^{+\ell} = \frac{\mu_0 I R}{4\pi} \hat{k} \left(\frac{\ell}{R^2(\ell^2+R^2)^{1/2}} - \frac{-\ell}{R^2(\ell^2+R^2)^{1/2}} \right)$$

$$= \frac{2\mu_0 I}{4\pi R} \hat{k} \left(\frac{\ell}{(\ell^2+R^2)^{1/2}} \right) = \frac{2\left(4\pi \times 10^{-7} \frac{T\cdot m}{A}\right)(10A)\hat{k}}{4\pi\,(0.10\,m)} \left[\frac{0.10\,m}{\left((0.10m)^2+(0.10m)^2\right)^{1/2}} \right]$$

$$= \frac{20 \times 10^{-7}}{(0.02\,m^2)^{1/2}} \hat{k}\,T = 1.41 \times 10^{-5}\,\hat{k}\,T$$

$$\vec{B}_{total} = 4 \cdot \vec{B}_1 = 4\left(1.41 \times 10^{-5}\,\hat{k}\,T\right) = \boxed{5.66 \times 10^{-5}\,\hat{k}\,T}$$