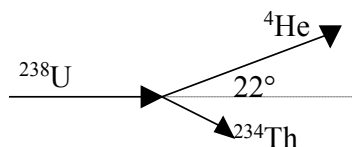


Chapter 9 Problem 50 †



Given

$$m_U = 238 \text{ AU}$$

$$m_{Th} = 234 \text{ AU}$$

$$m_{He} = 4.00 \text{ AU}$$

$$\vec{v}_U = 5.0 \times 10^5 \hat{i} \text{ m/s}$$

$$\vec{v}_{He} = 1.4 \times 10^7 \text{ m/s} \quad 22^\circ \text{ above the x-axis}$$

Solution

Find the velocity of the Thorium nucleus.

First convert the Helium velocity into rectangular coordinates.

$$\vec{v}_{He} = \{v \cos \theta \hat{i} + v \sin \theta \hat{j}\}$$

$$\vec{v}_{He} = \{(1.4 \times 10^7 \text{ m/s}) \cos(22^\circ) \hat{i} + (1.4 \times 10^7 \text{ m/s}) \sin(22^\circ) \hat{j}\}$$

$$\vec{v}_{He} = \{1.30 \times 10^7 \hat{i} + 5.24 \times 10^6 \hat{j}\} \text{ m/s}$$

From conservation of momentum, the momentum before the nucleus decays must be equal to the momentum after the nucleus decays. Therefore,

$$\vec{p}_{before} = \vec{p}_{after}$$

$$\vec{p}_U = \vec{p}_{Th} + \vec{p}_{He}$$

$$m_U \vec{v}_U = m_{Th} \vec{v}_{Th} + m_{He} \vec{v}_{He}$$

$$m_{Th} \vec{v}_{Th} = m_U \vec{v}_U - m_{He} \vec{v}_{He}$$

$$\vec{v}_{Th} = \frac{m_U \vec{v}_U - m_{He} \vec{v}_{He}}{m_{Th}}$$

$$\vec{v}_{Th} = \frac{(238 \text{ AU})\{5.0 \times 10^5 \hat{i} \text{ m/s}\} - (4.00 \text{ AU})\{1.3 \times 10^7 \hat{i} \text{ m/s} + 5.24 \times 10^6 \hat{j} \text{ m/s}\}}{(234 \text{ AU})}$$

$$\vec{v}_{Th} = \frac{\{1.19 \times 10^8 \hat{i}\} - \{5.2 \times 10^7 \hat{i} + 20.96 \times 10^6 \hat{j}\}}{(234)} \text{ m/s}$$

$$\vec{v}_{Th} = \frac{\{6.7 \times 10^7 \hat{i} - 20.96 \times 10^6 \hat{j}\}}{(234)} \text{ m/s}$$

$$\vec{v}_{Th} = \{2.86 \times 10^5 \hat{i} - 8.96 \times 10^4 \hat{j}\} \text{ m/s}$$

In polar coordinates the Thorium's velocity is

$$\vec{v}_{Th} = 3.0 \times 10^5 \text{ m/s} \quad \angle -17.4^\circ$$

†Problem from Essential University Physics, Wolfson