Chapter 9 Problem 48[†]

Given $m_1 = 1250 \ kg$ $\vec{v}_1 = \{36.2\hat{i} + 12.7\hat{j}\} \ m/s$ $m_2 = 448 \ kg$ $\vec{v}_2 = \{13.8\hat{i} + 10.2\hat{j}\} \ m/s$

Solution

If the car and wagon stick together, find their velocity after the collision.

By conservation of momentum, the momentum of the car-wagon system before and after the collision is the same.

$$\begin{split} \vec{p}_{before} &= \vec{p}_{after} \\ \vec{p}_1 + \vec{p}_2 &= \vec{p}_T \\ m_1 \vec{v}_1 + m_2 \vec{v}_2 &= (m_1 + m_2) \vec{v}_T \\ \vec{v}_T &= \frac{m_1 \vec{v}_1 + m_2 \vec{v}_2}{(m_1 + m_2)} \\ \vec{v}_T &= \frac{(1250 \ kg) \{36.2\hat{i} + 12.7\hat{j}\} \ m/s + (448 \ kg) \{13.8\hat{i} + 10.2\hat{j}\} \ m/s}{(1250 \ kg + 448 \ kg)} \\ \vec{v}_T &= \frac{\{45, 250\hat{i} + 15, 875\hat{j}\} \ kg \cdot m/s + \{6, 182\hat{i} + 4, 570\hat{j}\} \ kg \cdot m/s}{(1698 \ kg)} \\ \vec{v}_T &= \frac{\{51, 432\hat{i} + 20, 445\hat{j}\} \ kg \cdot m/s}{(1698 \ kg)} \\ \vec{v}_T &= \{30.3\hat{i} + 12.0\hat{j}\} \ m/s \end{split}$$

In polar coordinates this velocity is

$$v = \sqrt{(30.3 \ m/s)^2 + (12.0 \ m/s)^2} = 32.6 \ m/s$$
$$\theta = \tan^{-1} \left(\frac{12.0 \ m/s}{30.3 \ m/s}\right) = 21.6^{\circ}$$