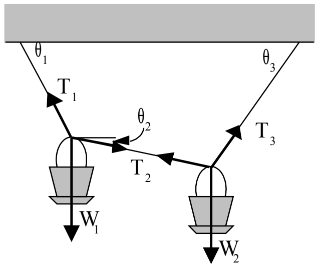
Chapter 5 Problem 73 †



Given
$m_1 = 3.85\ kg$
$m_2 = 9.28\ kg$
$\theta_1 = 54.0°$
$\theta_2 = 13.9°$
$\theta_3 = 68.0°$

Solution
Find the tension in each of the wires.

The free body diagrams for the two plants are drawn on the picture. The coordinate system is chosen to have the x-axis in the horizontal and the y-axis in the vertical. Write out Newton's 2^{nd} law for the first plant.

$$\Sigma \vec{F} = m\vec{a}$$

$$\vec{T_1} + \vec{T_2} + \vec{W_1} = m_1\vec{a}$$

The plant is stationary so there is no acceleration. The equation in unit vector notation is

$$-T_1 \cos\theta_1 \hat{\imath} + T_1 \sin\theta_1 \hat{\jmath} + T_2 \cos\theta_2 \hat{\imath} - T_2 \sin\theta_2 \hat{\jmath} - m_1 g \hat{\jmath} = 0$$

The x-component is then

$$-T_1 \cos\theta_1 + T_2 \cos\theta_2 = 0 \tag{1}$$

and the y-component is

$$T_1 \sin\theta_1 - T_2 \sin\theta_2 - m_1 g = 0 \tag{2}$$

Write out Newton's 2^{nd} law for the second plant.

$$\Sigma \vec{F} = m\vec{a}$$

$$\vec{T_2} + \vec{T_3} + \vec{W_2} = m_2\vec{a}$$

The plant is stationary so there is no acceleration. The equation in unit vector notation is

$$-T_2 \cos\theta_2 \hat{\imath} + T_2 \sin\theta_2 \hat{\jmath} + T_3 \cos\theta_3 \hat{\imath} + T_3 \sin\theta_3 \hat{\jmath} - m_2 g \hat{\jmath} = 0$$

The x-component is then

$$-T_2 \cos\theta_2 + T_3 \cos\theta_3 = 0 \tag{3}$$

†Problem from Essential University Physics, Wolfson

and the y-component is

$$T_2 \sin\theta_2 + T_3 \sin\theta_3 - m_2 g = 0 \tag{4}$$

Take equation (1) and solve for T_1.

$$T_1 = \frac{\cos\theta_2}{\cos\theta_1} T_2 \tag{5}$$

Substitute equation (5) into (2) and solve for T_2.

$$\frac{\cos\theta_2}{\cos\theta_1} T_2 \sin\theta_1 - T_2 \sin\theta_2 - m_1 g = 0$$

$$T_2(\cos\theta_2 \tan\theta_1 - \sin\theta_2) - m_1 g = 0$$

$$T_2 = \frac{m_1 g}{\cos\theta_2 \tan\theta_1 - \sin\theta_2} = \frac{(3.85\ kg)(9.8\ m/s^2)}{\cos(13.9^\circ)\tan(54.0^\circ) - \sin(13.9^\circ)}$$

$$T_2 = 34.4\ N$$

Substitute this result into equation (5) and get

$$T_1 = \frac{\cos(13.9^\circ)}{\cos(54.0^\circ)}(34.4\ N) = 56.8\ N$$

Now take equation (3) and solve for T_3.

$$T_3 = \frac{\cos\theta_2}{\cos\theta_3} T_2 = \frac{\cos(13.9^\circ)}{\cos(68.0^\circ)}(34.4\ N)$$

$$T_3 = 89.1\ N$$

None of the forces exceed 100 N; therefore, the fishing line will hold.