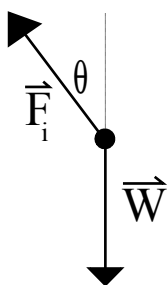


Chapter 5 Problem 42 †



Given

$$m = 45 \text{ kg}$$

$$r = 5.0 \text{ m}$$

$$v = 6.3 \text{ m/s}$$

Solution

a) Find the horizontal and vertical components of force exerted on the skate blades.

The free-body diagram is given above. Chose the x-coordinate to be to the right. Using Newton's 2nd law

$$\Sigma \vec{F} = m\vec{a}$$

$$\vec{F}_i + \vec{W} = m\vec{a}$$

Since the skater is going around a circle at constant speed the acceleration must be centripetal acceleration. The direction of this acceleration is in the -x direction. Write out the equation in unit vector notation.

$$-F_i \sin \theta \hat{i} + F_i \cos \theta \hat{j} - mg \hat{j} = -m \frac{v^2}{r} \hat{i}$$

The x-component of this equation is

$$-F_i \sin \theta = -m \frac{v^2}{r} \tag{1}$$

and the y-component of this equation is

$$F_i \cos \theta - mg = 0 \tag{2}$$

In equation (1) the horizontal component of the force on the skate blades is $F_i \sin \theta$. Solving for this quantity gives

$$F_h = F_i \sin \theta = m \frac{v^2}{r} = (45 \text{ kg}) \frac{(6.3 \text{ m/s})^2}{(5.0 \text{ m})} = 357 \text{ N}$$

The vertical component of the force on the skate blades is equal to the normal force which is the quantity $F_i \cos \theta$. From equation (2) this is

$$F_v = F_i \cos \theta = mg = (45 \text{ kg})(9.8 \text{ m/s}^2) = 441 \text{ N}$$

†Problem from Essential University Physics, Wolfson

b) Find the angle the skater is leaning without falling over.

The angle θ is the tangent of the opposite side (horizontal component) divided by the adjacent side (vertical component). This gives

$$\tan \theta = \frac{F_h}{F_v}$$

$$\theta = \tan^{-1} \left(\frac{F_h}{F_v} \right) = \tan^{-1} \left(\frac{357 \text{ N}}{441 \text{ N}} \right) = 39.0^\circ$$