Chapter 2 Problem 47 [†]

Given

$x = bt + ct^3$
$b = 1.50\ m/s$
$c = 0.640\ m/s^3$

Solution

a) Find the average velocity between 1.00 s and 3.00 s.

At $t_i = 1.00\ s$,

$$x_i = (1.50\ m/s)(1.00\ s) + (0.640\ m/s^3)(1.00\ s)^3 = 2.14\ m$$

At $t_f = 3.00\ s$,

$$x_f = (1.50\ m/s)(3.00\ s) + (0.640\ m/s^3)(3.00\ s)^3 = 21.78\ m$$

The average velocity is then

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i} = \frac{21.78\ m - 2.14\ m}{3.00\ s - 1.00\ s} = 9.82\ m/s$$

b) Find the average velocity between 1.50 s and 2.50 s.

At $t_i = 1.50\ s$,

$$x_i = (1.50\ m/s)(1.50\ s) + (0.640\ m/s^3)(1.50\ s)^3 = 4.41\ m$$

At $t_f = 2.50\ s$,

$$x_f = (1.50\ m/s)(2.50\ s) + (0.640\ m/s^3)(2.50\ s)^3 = 13.75\ m$$

The average velocity is then

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i} = \frac{13.75\ m - 4.41\ m}{2.50\ s - 1.50\ s} = 9.34\ m/s$$

c) Find the average velocity between 1.95 s and 2.05 s.

At $t_i = 1.95\ s$,

$$x_i = (1.50\ m/s)(1.95\ s) + (0.640\ m/s^3)(1.95\ s)^3 = 7.671\ m$$

At $t_f = 2.05\ s$,

$$x_f = (1.50\ m/s)(2.05\ s) + (0.640\ m/s^3)(2.05\ s)^3 = 8.589\ m$$

The average velocity is then

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i} = \frac{8.589\ m - 7.671\ m}{2.05\ s - 1.95\ s} = 9.18\ m/s$$

[†] Problem from Essential University Physics, Wolfson

d) Find the instantaneous velocity at $t = 2.00\ s$.

$$v = \frac{dx}{dt} = \frac{d(bt + ct^3)}{dt} = b + 3ct^2$$

At $t = 2.00\ s$,

$$v = (1.50\ m/s) + 3(0.640\ m/s^3)(2.00\ s)^2 = 9.18\ m/s$$

Notice that as the interval decreases, it approaches the instantaneous velocity.