Chapter 13 Problem 52 †

Given

$T = 12\ s$
$M = 600\ g = 0.6\ kg$
$r = 30\ cm = 0.3\ m$

Solution

Find the mass of the valve stem.

Since the wheel is balanced except for the presence of the valve stem, the only force causing rotation is the force of gravity on the valve stem. We know the angular frequency for a physical pendulum is

$$\omega = \sqrt{\frac{mgl}{I}}$$

where l is the distance the force is applied from the pivot point and mg is the force that gravity exerts on the pendulum. Since the wheel is balanced except for the valve stem, the only force causing the wheel to turn is gravity acting on the mass of the valve stem. The distance this force is applied from the pivot point is the radius of the wheel, R. The moment of inertia in this equation is that for the whole wheel since it must all rotate as a unit. Calculating the moment of inertia for the wheel gives

$$I = MR^2 + mR^2$$

Substituting into the angular frequency formula gives

$$\omega = \sqrt{\frac{mgR}{(M+m)R^2}} = \sqrt{\frac{mg}{(M+m)R}}$$

The time period is related to the angular frequency by the formula

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{\frac{mg}{(M+m)R}}} = 2\pi\sqrt{\frac{(M+m)R}{mg}}$$

Now solve for the mass of the valve stem, m.

$$\frac{T}{2\pi} = \sqrt{\frac{(M+m)R}{mg}}$$

$$mg\left(\frac{T}{2\pi}\right)^2 = (M+m)R$$

$$m\left(g\left(\frac{T}{2\pi}\right)^2 - R\right) = MR$$

$$m\left(g\left(\frac{T}{2\pi}\right)^2 - R\right) = MR$$

$$m = \frac{MR}{\left(g\left(\frac{T}{2\pi}\right)^2 - R\right)} = \frac{M}{\left(\frac{g}{R}\left(\frac{T}{2\pi}\right)^2 - 1\right)}$$

$$m = \frac{(0.60\ kg)}{\left(\frac{(9.8\ m/s^2)}{(0.30\ m)}\left(\frac{12\ s}{2\pi}\right)^2 - 1\right)} = 0.0051\ kg$$

$$m = 5.1\ g$$

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†Problem from Essential University Physics, Wolfson