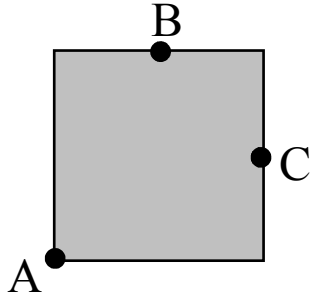## Chapter 12   Problem 15 [†]

Solution

Find the gravitational torque about each of the indicated points on a square plate of uniform thickness.

The gravitational force is always downward from the center of the plate with a value of

$$\vec{F_g} = -mg\hat{j}$$

The position vector of the center of gravity with respect to point A is

$$\vec{r} = \{\tfrac{1}{2}l\hat{i} + \tfrac{1}{2}l\hat{j}\}$$

The gravitational torque about point A is

$$\vec{\tau} = \vec{r} \times \vec{F}$$

$$\vec{\tau} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \tfrac{1}{2}l & \tfrac{1}{2}l & 0 \\ 0 & -mg & 0 \end{vmatrix}$$

$$\vec{\tau} = -\tfrac{1}{2}mgl\hat{k}$$

The position vector of the center of gravity with respect to point B is

$$\vec{r} = \{-\tfrac{1}{2}l\hat{j}\}$$

The gravitational torque about point B is

$$\vec{\tau} = \vec{r} \times \vec{F}$$

$$\vec{\tau} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & -\tfrac{1}{2}l & 0 \\ 0 & -mg & 0 \end{vmatrix}$$

$$\vec{\tau} = 0$$

The position vector of the center of gravity with respect to point C is

$$\vec{r} = \{-\tfrac{1}{2}l\hat{i}\}$$

The gravitational torque about point C is

$$\vec{\tau} = \vec{r} \times \vec{F}$$

$$\vec{\tau} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -\tfrac{1}{2}l & 0 & 0 \\ 0 & -mg & 0 \end{vmatrix}$$

$$\vec{\tau} = \tfrac{1}{2}mgl\hat{k}$$

[†]Problem from Essential University Physics, Wolfson