Chapter 7 Problem 50 [†]

Given
$a = 4.04 \times 10^{-28}$
$b = 5.52 \times 10^{-98}$
$n = 8.22$

Solution
Find the equilibrium separation between the ions in NaCl.

The formula for determining the potential energy between ions is

$$U = \frac{b}{r^n} - \frac{a}{r} = br^{-n} - ar^{-1}$$

The force between the ions is the negative derivative of the potential energy function.

$$F = -\frac{\partial U}{\partial r}$$

At equilibrium the force is zero. Therefore,

$$0 = F = -\frac{\partial U}{\partial r} = -\left\{-nbr^{-n-1} - (-1)ar^{-2}\right\} = \frac{nb}{r^{n+1}} - \frac{a}{r^2}$$

Solving for r gives

$$\frac{a}{r^2} = \frac{nb}{r^{n+1}}$$

$$\frac{r^{n+1}}{r^2} = \frac{nb}{a}$$

$$r^{n-1} = \frac{nb}{a}$$

$$r = \left(\frac{nb}{a}\right)^{1/(n-1)}$$

Substitute in the provided values gives

$$r = \left(\frac{(8.22)(5.52 \times 10^{-98})}{4.04 \times 10^{-28}}\right)^{1/(8.22-1)} = (1.123 \times 10^{-69})^{0.1385} = 2.82 \times 10^{-10}\ m$$

[†]Problem from Essential University Physics, Wolfson