Chapter 7 Problem 21 [†]

Given
$m = 120\ g = 0.120\ kg$
$k = 430\ N/m$
$\Delta x = 71\ cm = 0.71\ m$

Solution
Find the maximum height that the arrow reaches.

The bow acts like a spring. Since the bow is drawn a distance of Δx, the potential energy stored in the bow is

$$\Delta U = -W = -\int_{x_0}^{x_f} -kx dx = \frac{kx_f^2}{2} - \frac{kx_0^2}{2}$$

Assuming that no energy is stored in the bow when $x_0 = 0\ m$, then the potential energy is

$$\Delta U = \frac{kx_f^2}{2} \tag{1}$$

When the bow is fired, this potential energy is converted to kinetic energy and the arrow moves upward. As the arrow moves upward, its kinetic energy is converted to gravitational potential energy. The potential energy of gravity is

$$\Delta U = -W = -\int_{y_0}^{y_f} -mg dy = mg(y_f - y_0)$$

Assuming that the zero gravitational potential energy level is when $y_0 = 0$, then the gravitational potential energy is

$$\Delta U = mgy_f \tag{2}$$

Setting equations (1) and (2) equal to each other gives a value for y_f of

$$\frac{kx_f^2}{2} = mgy_f$$

$$y_f = \frac{kx_f^2}{2mg} = \frac{(430\ N/m)(0.71\ m)^2}{2(0.120\ kg)(9.80\ m/s^2)} = 92.2\ m$$

[†]Problem from Essential University Physics, Wolfson