

Chapter 6 Problem 79 †

Given

$$y = ax^2 - bx$$

$$a = 2 \text{ m}^{-1}$$

$$b = 4$$

$$\vec{F} = cxy\hat{i} + d\hat{j}$$

$$c = 10 \text{ N/m}^2$$

$$d = 15 \text{ N}$$

Solution

Find the work when going from $x = 3 \text{ m}$ to $x = 6 \text{ m}$.

Since we are working in 2 dimensions, the value of the differential displacements is $d\vec{r} = dx\hat{i} + dy\hat{j}$. For work the dot product becomes

$$W = \int_{r_1}^{r_2} \vec{F} \cdot d\vec{r} = \int_{r_1}^{r_2} (F_x dx + F_y dy) = \int_{r_1}^{r_2} (cxy dx + d dy) \quad (1)$$

Substitute in for y with the relationship $y = ax^2 - bx$ and for dy the relationship $dy = (2ax - b)dx$. This relationship comes from taking the derivative of the function for y .

$$\frac{dy}{dx} = \frac{d(ax^2 - bx)}{dx} = 2ax - b$$

Then multiply both sides by dx to get this relationship. Now substitute for y and dy in the work equation (1).

$$W = \int_{r_1}^{r_2} (cxy dx + d dy) = \int_{x=0}^{x=3} (cx(ax^2 - bx) dx + d(2ax - b) dx) = \int_{x=0}^{x=3} (cax^3 - cbx^2 + 2dax - db) dx$$

Notice that after this substitution the integral only depends on x and, therefore, the limits of integration only depend on x .

Now perform the integration with respect to x .

$$W = \left| \frac{cax^4}{4} - \frac{cbx^3}{3} + \frac{2dax^2}{2} - dbx \right|_{x=0}^{x=3}$$

Substituting in the values for a , b , c , and d and solving gives

$$W = \left(\frac{(10)(2)x^4}{4} - \frac{(10)(4)x^3}{3} + \frac{2(15)(2)x^2}{2} - (15)(4)x \right) \Big|_{x=0}^{x=3}$$

$$W = \left(5x^4 - \frac{40x^3}{3} + 30x^2 - 60x \right) \Big|_{x=0}^{x=3}$$

$$W = \left(5(3)^4 - \frac{40(3)^3}{3} + 30(3)^2 - 60(3) \right) - \left(5(0)^4 - \frac{40(0)^3}{3} + 30(0)^2 - 60(0) \right)$$

$$W = (405 - 360 + 270 - 180) - 0 = 135 \text{ J}$$

†Problem from Essential University Physics, Wolfson